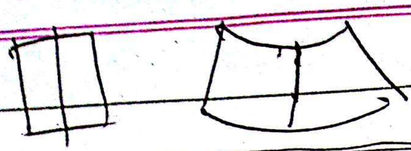


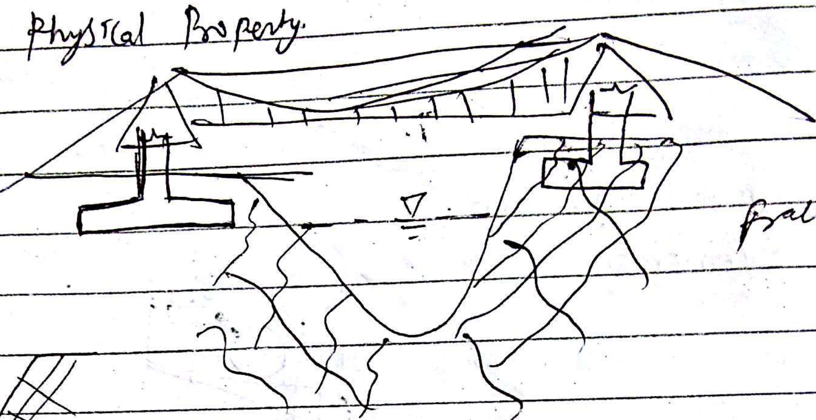
"Rock Engineering"

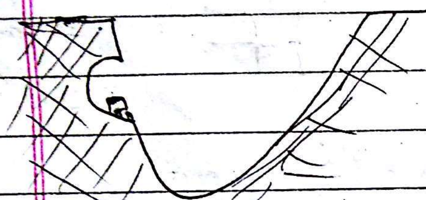
→ Introduction:-

→ Stereographic projection

→ Engineering and Physical Property.

(linear and non linear equations.)





UCS- uniaxial compressive strength

Indirect Methods of getting UCS

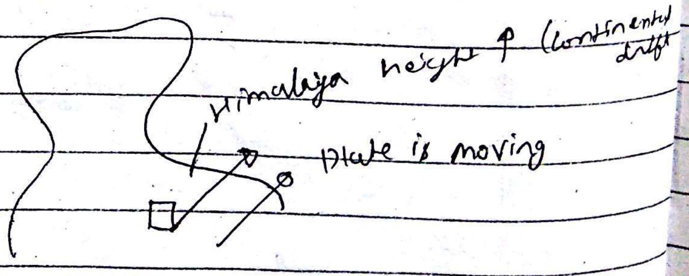
- determination of Tensile Strength

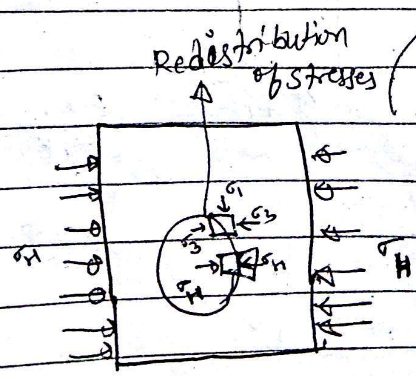
- Tri-axial Test

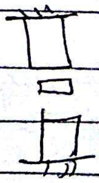
$$\sigma_{1f} = f(\sigma_3)$$

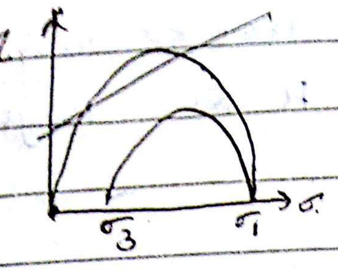
$$\tau_f = f(\sigma_n)$$

→ Strength Criteria.









Prior to excavation

after excavation $\sigma_3 = 0$

σ_1 (let say it is same)

Engineering Behaviour of Rocks Mass :-

Soil Mass :- We tested soil mass specimens

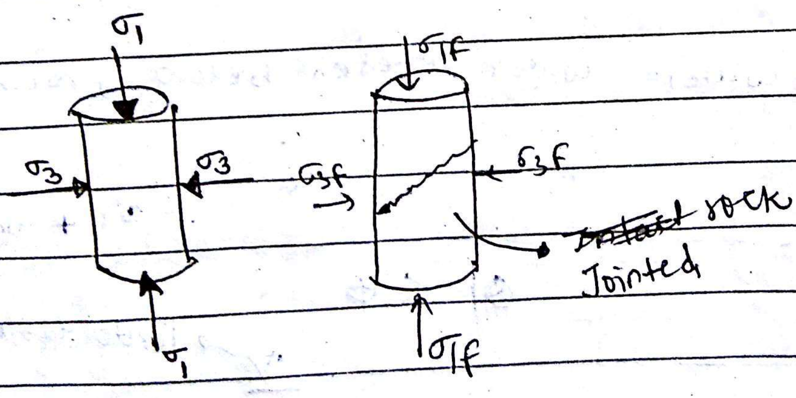
Rock Mass :- Not possible to Rock Mass specimens.

⇒ Start with intact rock

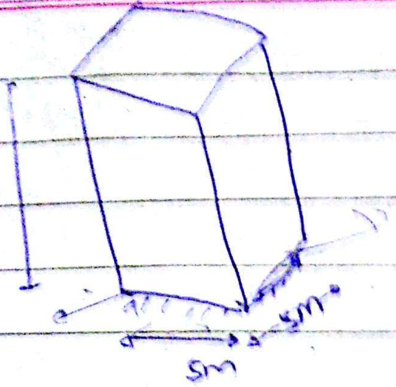
⇒ then scale it down.

⇒ Incorporate effect of fracture

Rock Mass = f(intact rock + fracture)



$\gamma = 26\ kN/m^3$



$E = 20\ MPa$

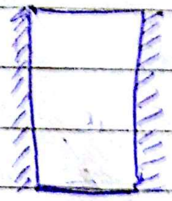
$\nu = 0.2$

① Plot variation of vertical stress due to self weight with depth

② Plot variation of vertical strain and lateral strain with depth (consider base frictionless)

* Comment what will happen if base is not frictionless.

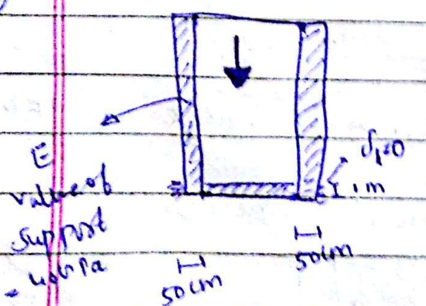
③



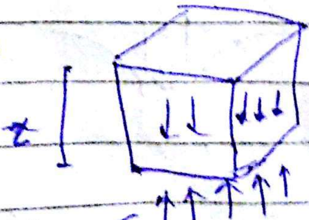
Rigid support in one direction. (frictionless)

* will there be any stress in the support (- Horizontal stress)

④

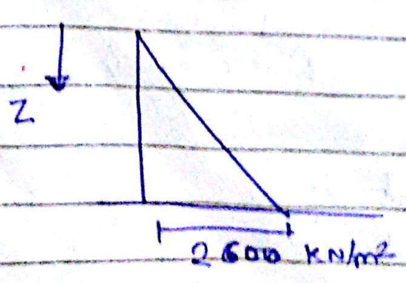


Answer :- ①



$\sigma_z (5\times5) = 26\,(z)(5\times5)$

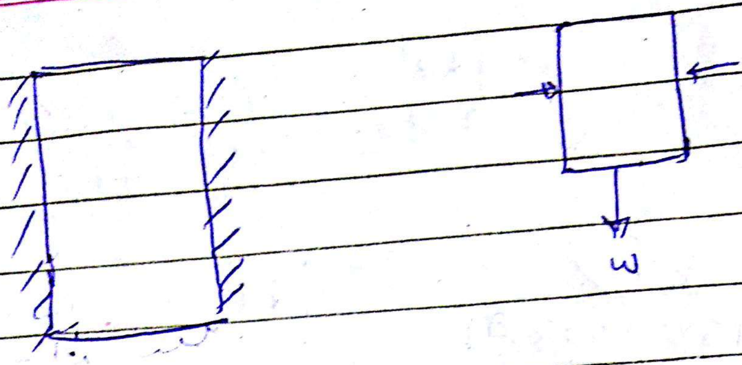
$\boxed{\sigma_z = 26z}$



vertical stress due to self load

depth z from Top.

Figure ③



Horizontal Stress = $-\nu\sigma_z$ ⇒ $-(0.2)(26z)$ ⇒ $-5.2z$

Next TW class:

A-4 paper = 4-5

board pin , eraser, Sharpner, Pencil, Pen.

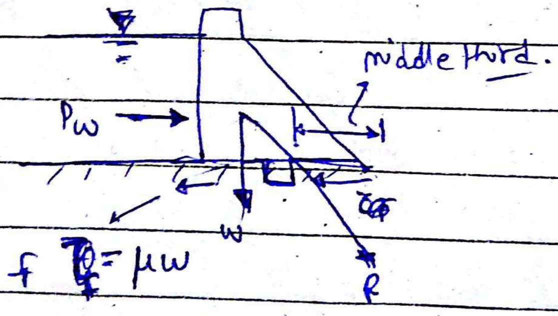
Field Applications:-

1960 s ⇒ " Two failure"

Malpasset Dam ⟹ Rock Mech

Vajont Reservoir

① Large Dam



failure at:

Toppling

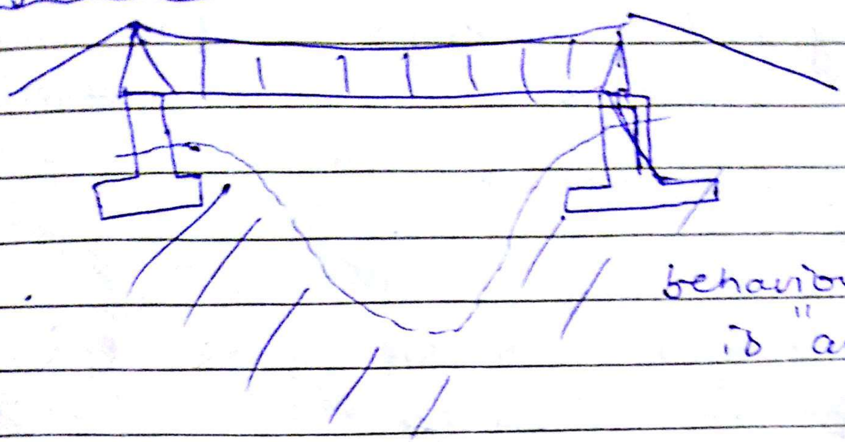
Sliding

Discontinuity ⇒ Joints / fracture

Continuous Joint is dangerous.

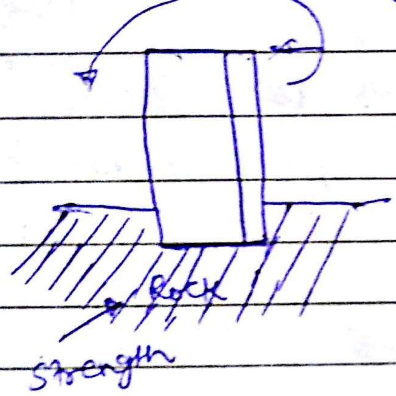
* Systematic and irregular Joints.

② Large Bridges:-



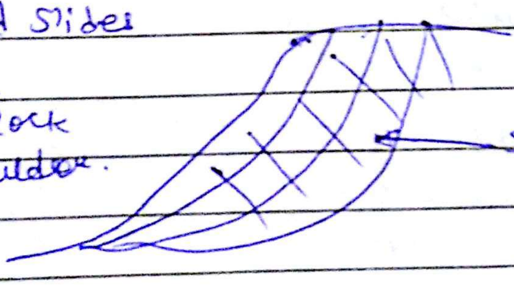
behaviour of mass is "anisotropic".

③ High Rise Buildings



④ Land Slides

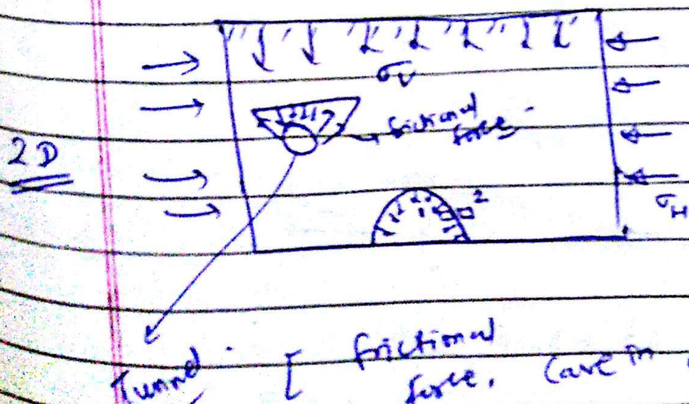
* Complex = Soil + Rock + Boulder.



strength along the failure surface.

⑤ Tunnels:-

* stress induced Instability

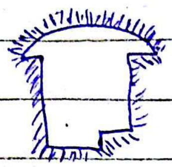


1, 2 (equilibrium before excavation)

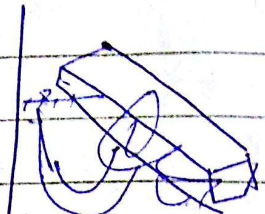
after excavation, 2 will be unstable.

[frictional force, cave in होने से बचाएगा)

⑥ Underground Power Houses . [extremely difficult structure, difficult to construct]



analyze in 3D
→ size
→ not analysed in plane strain case



Terzaghi theory does not valid at ends

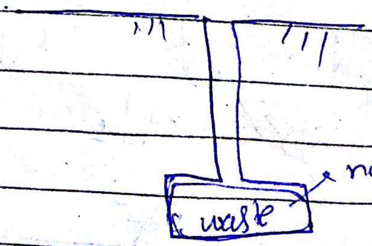
* Structural Instability

* Tunnel is analysed through Plane strain Case.

⑦ * Defence

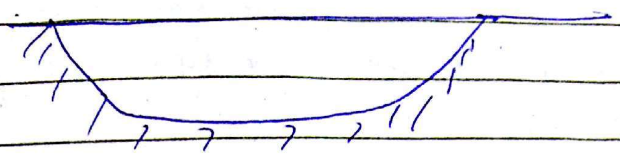
① Underground utilities.

⑧ Nuclear Power Plants
- Very sensitive
- "waste"
- Nuclear waste disposal ["dispose underground"]



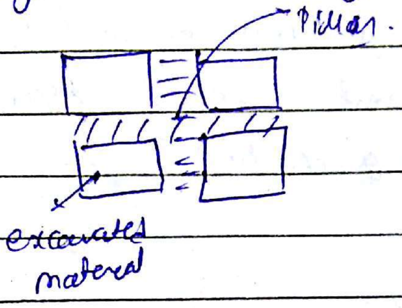
nuclear waste (Protective measures)

⑨ Open Cast Mining
- Huge excavation (big pit)
* Stability of Slopes is concerned



* ~~Giv.~~ CE Slope Stability

(ii) underground Mining :-



* mining slope (FOS =1)
* failure with own weight

Definitions

Rock Mechanics :- It is the theoritical and applied science of the mechanical behaviour of the rock. It is that branch of mechanics which is concerned with response of the rock to the ~~rock~~ force of its physical environment.

Rock :- It is a Competent materials. It is a aggregate of minerals.

$\boxed{\sigma_{ci} > 1\,\text{MPa}}$ If $\sigma_{ci} > 1$ mpa then it is rock.

* High strength and low strain than soil

17/1/17

Date : ___/___/___

Rock Engineering

Definitions / Terminology

① Rock Mechanics

② Rock

③ Micro fissure - Very fine crack in the rock.

Always exist in rock at boundary of Crystals.

- width $< 10^{-6}$ m
- width $< 1\ \mu$ (0.001 mm)
- length $\simeq$ 1 to 2 Crystals

④ Microfracture

May exist in Intact Rock

- Little greater than Micro fissure
- width $\simeq$ 0.01mm
- Not visible to naked eyes

⑤ Microfracture

wider than $\simeq$ 0.1mm

Very large length (tens of meters)

Exist in Rock Mass

not in Intact Rock

⑥ Intact Rock :-

Individual Rock specimen free from any flaw (discontinuity)

⑦ Fault :- It is a fracture in the rock along which, displacements have already taken place. Displacements ~~has~~ may be Very large, there is a zone where large displacements might occur in the past. It is very difficult to ~~est~~ measure the ~~measure~~ width of the fault zone. Because of large displacements, material is crushed in walls. walls sinks inside.

Slicken Side

Gouge Material is formed. Shear strength along this plane will be very low depending on the amount of displacements and type of gouge material formed.

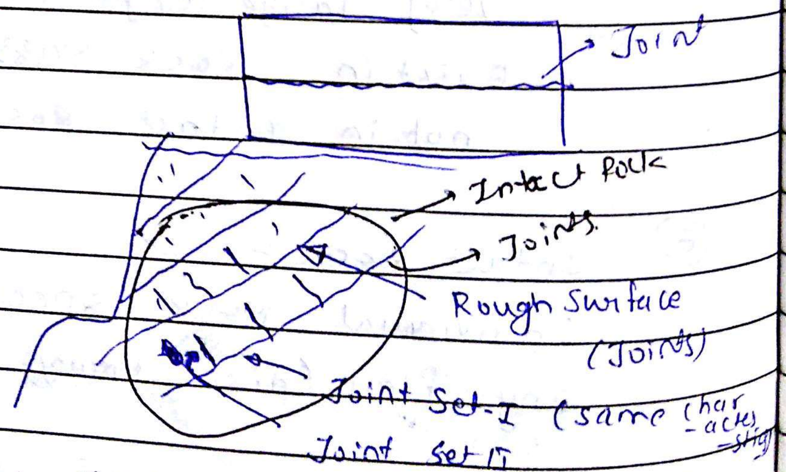
Powdered material of crushed walls.

* large displacement → Gouge material fine
* small displacement → Gouge material course

⑧ Joint :-

It is a fracture along which there has been no displacements or almost nill displacement.

* Generally Joints will be occurring in sets. A number of sets will form Joints System.



Joint System = 2 Joint Sets + Random joints.

⑨ Rock Mass

Large mass where blocks of intact Rock are separated by discontinuity.

Large Mass (intact Rock + joints)

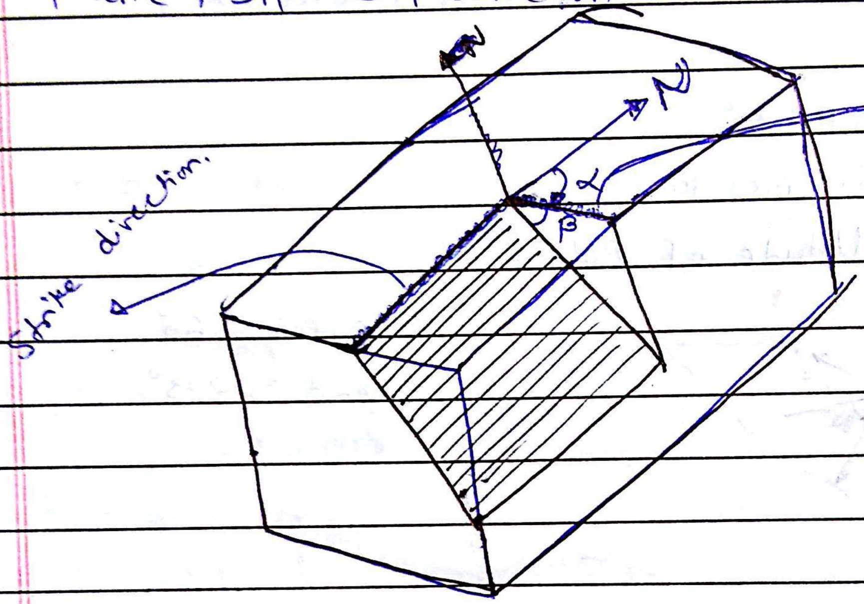
Date : ___/___/___

Stereographic Projections:-

Joints/"plane" (only angles will be considered)

How to present - Planner features graphically.

plane ; Dip ; Dip direction → Altitude of a plane.



Angle in horizontal Plane ⇒ w.r.t N (WCB) (whole Circle Bearings)

β = **Dip** ⇒ Angle the plane makes with Horizontal Plane.

* Slope of line of steepest gradient.

* Dip direction :- ~~Angle to which Horizontal Plane is making w.r.t North~~

* line ⊥r to strike direction.

Strike direction:- Common line b/w Horizontal plane and fracture plane.

UPCP

Pole is "Normal to the plane"

Plunge - Angle of Pole with respect to Horizontal

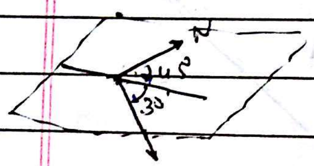
Trend - w.r.t North.

+ Plan of plunge line w.r.t North direction

Q. dip = 30°

dip direction = 45°

altitude of Pole :-



Plunge :- 60°

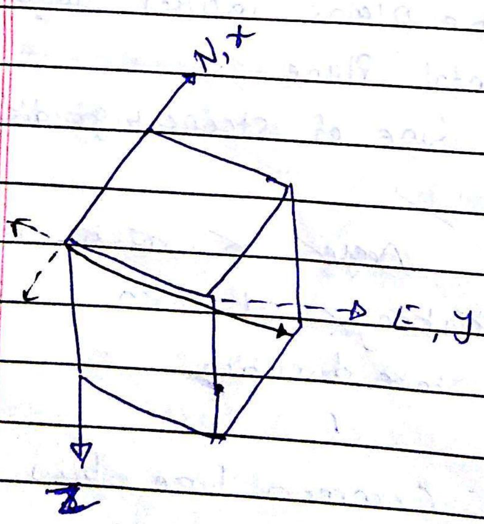
Trend = 225°

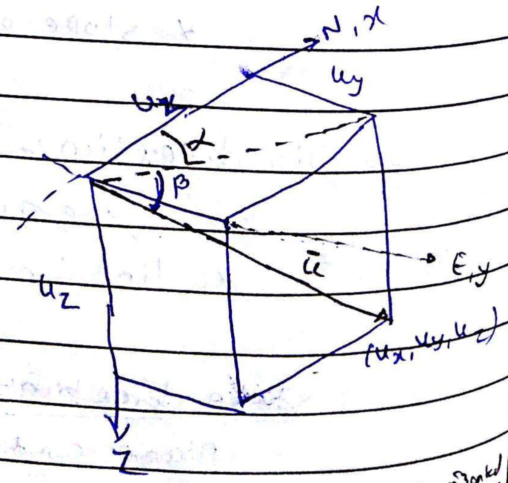
Mathematics Representation :-

Planes/Lines ⇒ Angles

3D :- Plane :- Altitude → xy° / ABC°

⇒ 05 / 010°





Clockwise direction { Trend is α → measured in Horizontal plane; Plunge is β → measured in vertical plane.

Trend = $\tan\alpha = \dfrac{u_y}{u_x}$, $\alpha = \tan^{-1}\left(\dfrac{u_y}{u_x}\right) + Q$

If $\alpha = 0-90$; $u_x \to +ve$, $u_y \to +ve$, $Q = 0$

If $\alpha = 90°-180°$, $u_x = -ve$, $u_y = +ve$, $Q = +180°$

If $\alpha = 180°-270°$, $u_x = -ve$, $u_y = -ve$, $Q = 180°$

If $\alpha = 270°-360°$, $u_x = +ve$, $u_y = -ve$, $Q = 360°$

plunge $\Rightarrow \tan\beta = \dfrac{u_z}{\sqrt{u_x^2+u_y^2}} \Rightarrow \beta = \tan^{-1}\left(\dfrac{u_z}{\sqrt{u_x^2+u_y^2}}\right)$

$u_x = |\bar{u}|\cos\beta\cos\alpha$, $u_y = |\bar{u}|\cos\beta\sin\alpha$

$u_z = |\bar{u}|\sin\beta$

direction cosine: $l = \dfrac{u_x}{|\bar{u}|} \Rightarrow \cos\beta\cos\alpha$

$m = \dfrac{u_y}{|\bar{u}|} \Rightarrow \cos\beta\sin\alpha$

$n = \dfrac{u_z}{|\bar{u}|} \Rightarrow \sin\beta$

'β' is always +ve

Consider always, n to be +ve i.e. β is always +ve

direction Ratio $\Rightarrow l : m : n$

If it is given $1 : -2 : -3$

then consider it as $-1 : 2 : 3$ ✓

Example

Plane altitude $\Rightarrow 30° | 110°$

Normal $\Rightarrow 60° | 290°$

UPCP

Example Normal to the plan is having direction ratios ~~1~~1, - 2, -3 .

$\Rightarrow$ -1, 2, 3

$\sin\beta = \frac{3}{\sqrt{1^2+2^2+3^2}} = \frac{3}{\sqrt{14}}$, $\beta = \sin^{-1}\left(\frac{3}{\sqrt{14}}\right)$

si

Stereographic Projection :-

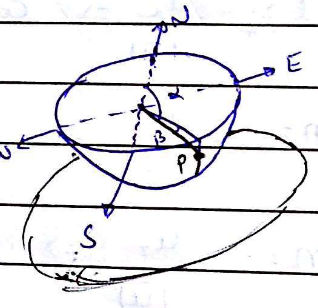
Graphical presentation of angles (No distances)

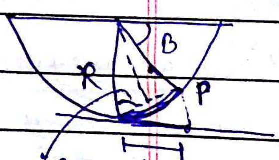
equal area projection

Line lower Hemisphere

assumption

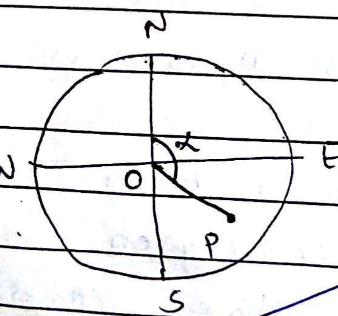
- every Line is passing through center





$\frac{90+\beta}{2}$ $r' = 2R\cos\left(\frac{90+\beta}{2}\right)$

= chord distance



'OP' is measure of 'β'

$OP = r' = 2R\cos\left(\frac{90+\beta}{2}\right)$

$r'' =$ Just to make simple r' is divided by $\sqrt{2}$

for $\beta = 0$

$r' = \sqrt{2}R$

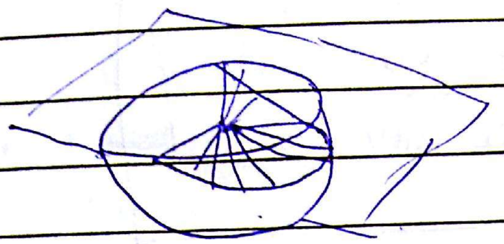
$r'' = \sqrt{2}R\cos\left(\frac{90+\beta}{2}\right)$

So, they divided by $\sqrt{2}$

finally $\boxed{r'' = \sqrt{2}R\cos\left(\frac{90+\beta}{2}\right)}$ ताकि equal area है।

Plane

assumption:- every plane is Passing through center



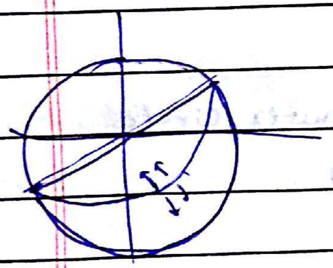
Intersecting Plane will be a circle.

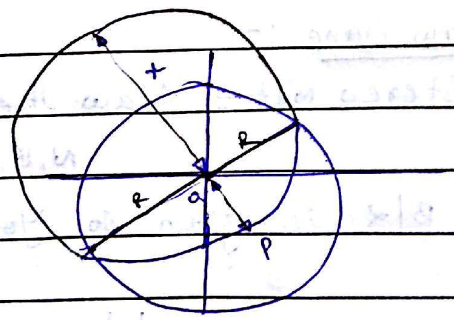
* Intersecting plane will have many number of line Passing through center

If plane attitude is having α = dip direction
β = dip

Trend of line of steepest slope = α = dip direction,
Plunge of line of steepest slope = β = dip

* stereographic projection of intersecting circle will also be a part of circle.





$$x\left[R\sqrt{2}\cos\left(\frac{90+\beta}{2}\right)\right] = R(R)$$

$$\Rightarrow \quad x = \frac{R}{\sqrt{2}\cos\left(\frac{90+\beta}{2}\right)}$$

Radius of bigger circle $\Rightarrow$ $\cancel{R}$ $\dfrac{x + R\sqrt{2}\cos\left(\frac{90+\beta}{2}\right)}{2}$

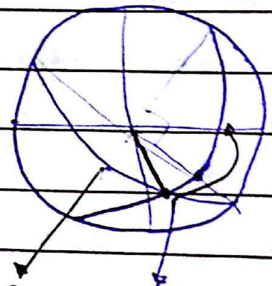
Radius $= \dfrac{2R + 2R\cos^2\left(\frac{90+\beta}{2}\right)}{2\sqrt{2}\cos\left(\frac{90+\beta}{2}\right)}$

$$\boxed{R_0 = R\left[\frac{1 + 2\cos^2\left(\frac{90+\beta}{2}\right)}{2\sqrt{2}\cos\left(\frac{90+\beta}{2}\right)}\right]}$$

Example P_1 - Plane attitude 30/120

$R = 3\,cm$

P_2 - Plane attitude = 45/240



P_2 P_1

To draw Plane :-

(1) Stereo Net ⇒ draw the outer circle

N, E, S, W

(2) β / α is given to you

(3) Mark Point 'P' at angle α to the north direction.

(4) Rotate the circle and bring the Point 'P' either on East or west direction.

Because we can measure dip only in East and west direction. from periphery.

(5) And draw the great circle.

Date : ___/___/___

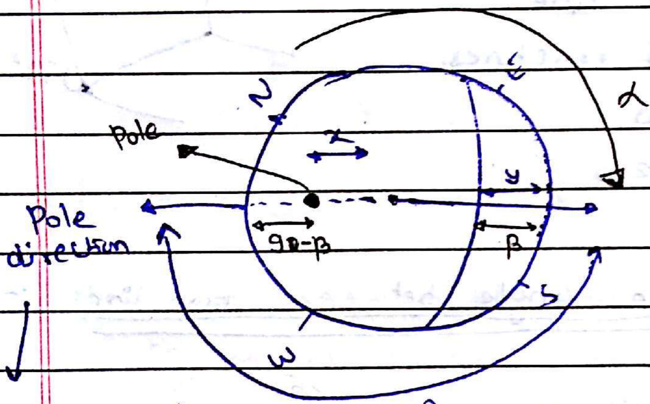
* Pole is the normal to the Plane.

To draw Pole

suppose attitude of Plane 30/120

attitude of Pole → 60/300

(1) Mark the Point 'P' at 300°.
(2) Rotate the point 'p' and bring the Point 'p' either on 'East or west direction', mark the point Corresponding to dip.
(3) Measure dip from periphery.



$\boxed{x = y = \beta}$

Trend of Pole $= 180 + \alpha$

Plunge of Pole → $90 - \beta$

To get line of Intersection of Plane :-

Two Planes are given
30/120
50/200

Plot two planes on stereonet and marked the intersection between these two planes. you will get a point that will be a line.

(UPCP)

* Mark the pole and rotate the P_1 and P_2 in such a way that both lies on great circle.
* So pole of that great circle will be the line of intersecting.

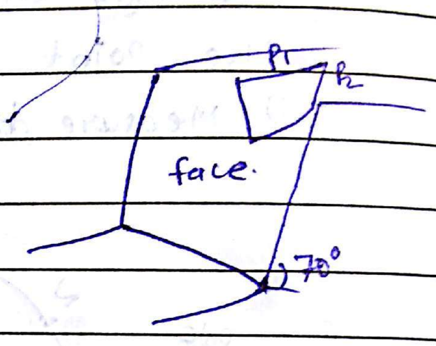
Problem :-

Slope face = $70^\circ/135^\circ$

P_1 and P_2 are Plane of fractures.

P_1 = 30/120

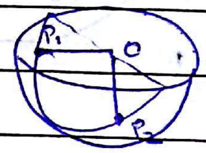
P_2 = 50/200

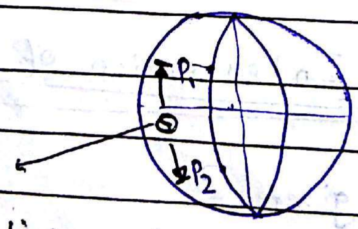


To determine angle between two lines :-

P_1 and P_2 will lie on Same plane (same great circle)

OP_1 and OP_2 are two lines,





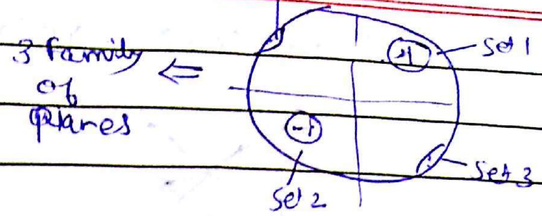
angular distance 'θ' — this will be the angle b/w lines

How to get Potential Failure mode in field:-

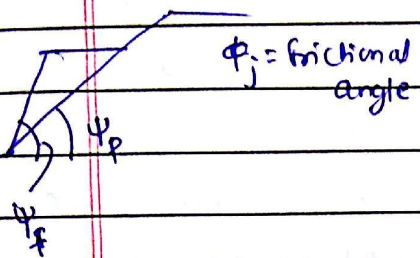
Field:- assume field attitude $30^\circ/120^\circ$

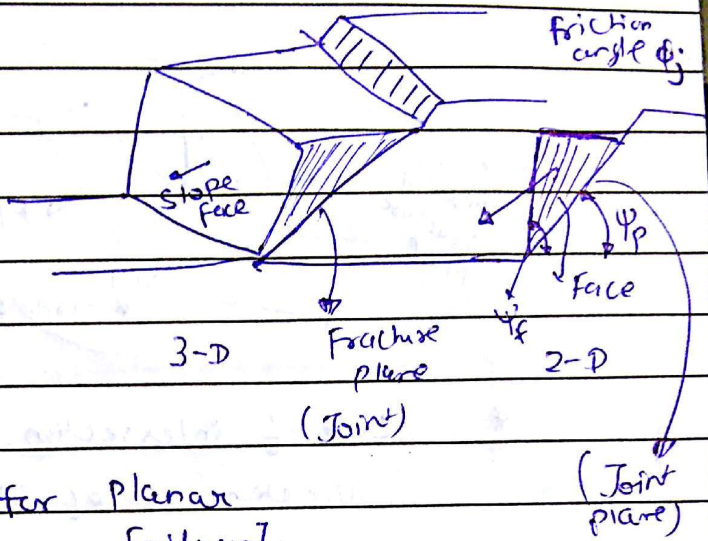
Pole - ...

Date : set 3 /



Planar Failure :-





If $\psi_f > \psi_p > \phi_j$ [for planar Failure]

* (slope face should be in same direction as Joint Plane)

<u>kinematic analysis of Rock Slope :-</u>

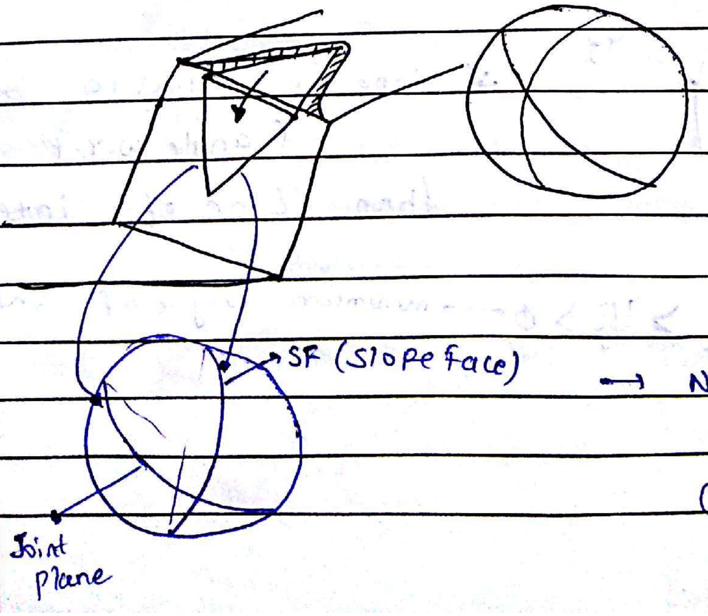
(Potential failure Mode)

a) Plane Failure

b) wedge Failure

c) Toppling failure

d) circular failure



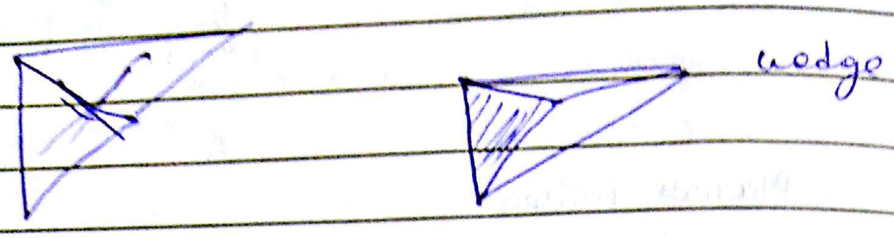
→ No Planner failure can possible

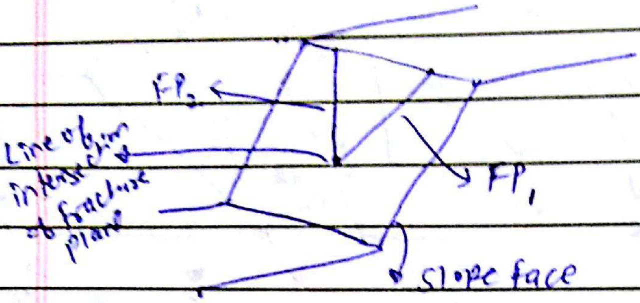
(not in same direction

(not ~~in~~ same dipping)

b) wedge failure

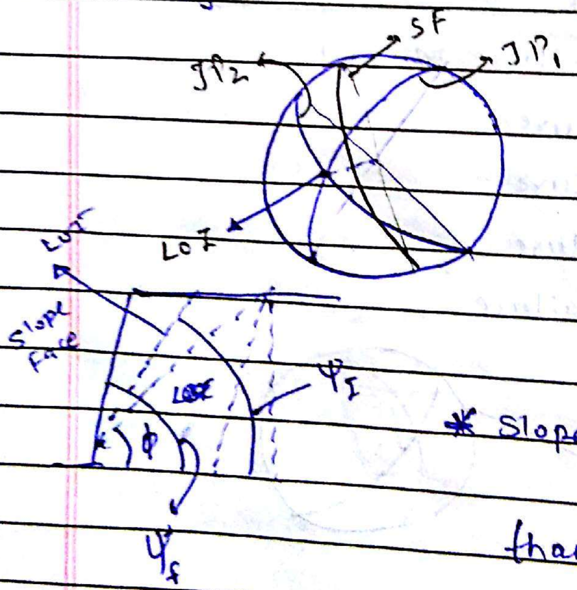
Above case represent wedge failure





* Line of intersection should be in same direction as of slope face. (Trend should be same)

* Line of intersection should be steeper than ϕ_j



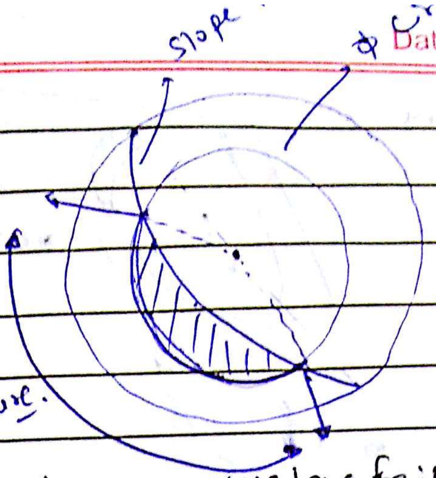
Line of Intersection and Slope Face is having same Trend. (MOST Ideal Case)

* Slope Face has to be steeper (angle w.r.t Horizontal) than Line of intersection.

$\psi_f > \psi_I > \phi$ → minimum angle of Line of intersection

Date : / /

Example

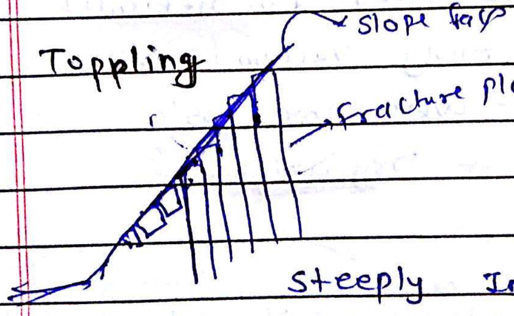


Steps for Determing wedge failure

1. Draw slope face
2. Measure distance ф (Plunge) and draw a circle passing through that point.
3. Get the ~~line of intersection~~ Shaded area. (area b/w ф circle and slope face)
4. Get the Line of intersection and if this line lies in the shaded area than there will be possibility of wedge failure. [line of intersection of two Joint Faces]

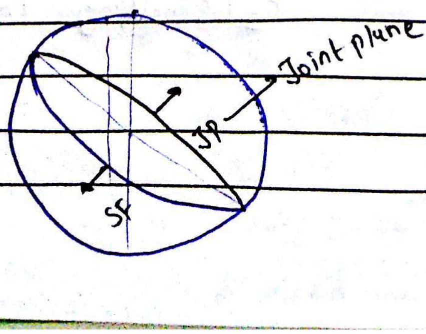
$\boxed{\psi_F > \psi_I > \phi}$ for wedge failure

C) Toppling



steeply Inclined Joint planes towards the slope face causes toppling. ~~of~~

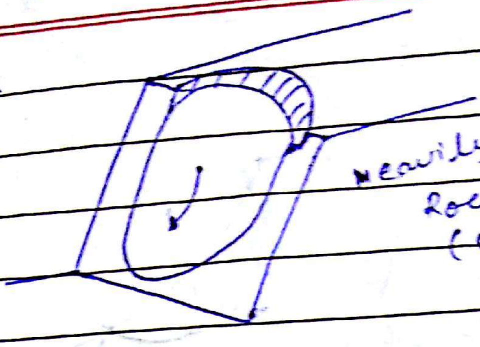
Case of Toppling →



(Joint plane Trend will be Somewhat opposite to slope face but having more dip angle than slope face.

UPCP

d) Circular

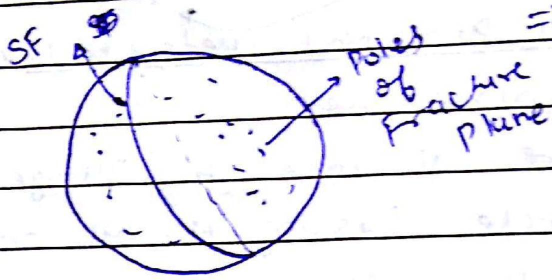


Heavily fractured Rock mass (~~sides~~

| fracture plane is in every direction

(no family of fractures

⇒ Random fracture.



Intact Rocks :-

1. Physical and engineering Properties :-

Index Properties :-

Soils → Porosity / Void Ratio

$$\boxed{\text{Void index} = \frac{\text{wt. of water absorbed}}{\text{wt. of dry Rock}}}$$

— * sp is saturated (v. Difficult)

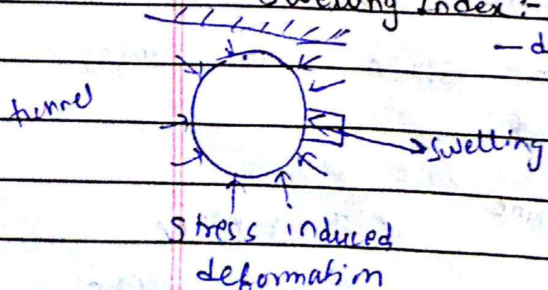
Wipe Surface wt → — Apply vaccum / boil

— Over 12 hr – 105°c

— Dry weight.

Swelling Index :-

— due to Presence of Clay mineral



ε_s (swelling index) $= \frac{\Delta l}{l}$

$= \frac{\text{Change in Dimension length}}{\text{Original dimension}}$

Hydraulic Permeability :-

depends on Fractures, Interconnected fractures $\Leftarrow$

$$q = k i A$$

$$q = k \frac{dh}{dx} \cdot A$$

$k \downarrow$ Hydraulic ~~Permea~~ Conductivity (Properties of medium and fluid)

=) Pressure Development Phenomenon

$$q \propto k \qquad q \propto \frac{1}{\mu}$$

$$q = \frac{k}{\mu} \cdot \frac{dh}{dx} \cdot A$$

where k = Hydraulic Permeability (m^2) ↓ (Properties of medium)

μ = Viscosity (Permeant) ($N \cdot s/m^2$) (Fluid Property)

$$\boxed{1 \text{ Darcy} = 9.86 \times 10^{-9} \ cm^2}$$ → for Hydraulic Permeability

Lab Tests

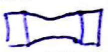
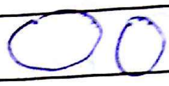
- Chemical Composition
  - ↓ Minerals
- Physical Property Tests
  - ↓
  - * Density
  - * Sp. gravity
  - * Void Ratio / Porosity
  - * Swelling Index
  - * Void index
- Engineering Property (Analysis and Design)
  - Static ~~(often used)~~
    - Time dependent
      - * Creep
      - * Relaxation
    - Time Independent
      - * Compression and
      - * Tension
      - * DST, ATBx
  - Dynamic ~~(special~~ cases)

Date : 02 | 02 | 2017

Specimen :-

* Definate shape
* It may be of Irregular Shape

Specimen

- Regular
  - Cylinder (plates, bars, blocks)
  - Cubic
  - Prismatic
- Special shapes
  - 
  - Briquette
- Irregular
  - 
  - Sub Rounded
  - 
  - platy

Sample :-

- Statistical quantity

Rocks :: wide scatter in Results.

Preparation = STRINGENT

For getting Result we need to use Probabilistic approach (ROCKS is uncertain material) & ⓘ

Drilling Sizes :-

EX	— 7/8"	≃ 23 mm
AX	— 1 1/8"	≃ 28 mm
BX	— 1 5/8"	≃ 41 mm
$\boxed{\text{NX}}$	— 2 1/8	≃ 54 mm → for Projects (minimum dia of specimen) → while Testing.

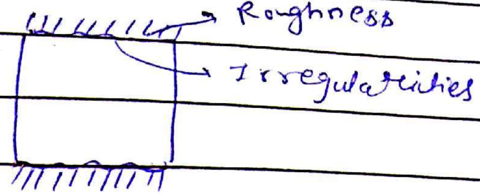
L/D Ratio for Regular specimen :-

		L/D
Compression	—	2.0 – 3.0
Bending	—	3 – 7
(ब्राजिलियन) ⓘ → Bragilian	—	0.5 to 1.0
Punch Test (Disc)	—	0.2 to 0.25

Date : ___/___/___

Tolerance limits :-

Compression Test :-

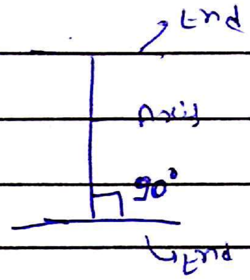


first Tolerance limit

(a) ends should be Smooth to 0.02 mm or 0.002D

$\underbrace{0.02\text{ mm or } 0.002D}_{\text{whichever is minimum}}$

(b)

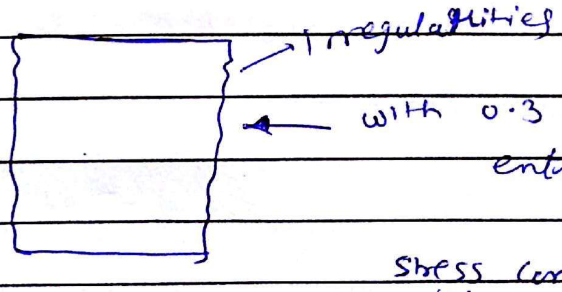


deviation in angle should be less than 0.001 radian

. It should be perfectly 90°

if not then load will be not passing through axis. Some ecentricity will comes in the account so moment and Bending will occur

(c)



with 0.3 mm over the entire length.

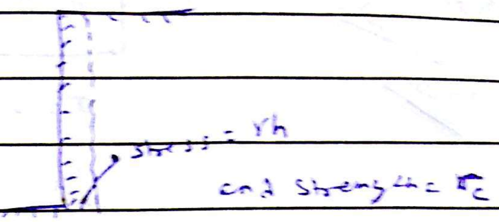
Stress concentration will occur due to irregularities resulting premature failure.

Intact Rock Properties:-

UCS:-

Probabilistic analysis:-

when $\gamma h < \sigma_c$ (safe)



assume γh value is 100% correct

σ_c is having some scattering (not 100% sure)

✱ so, what is the Probability that $\sigma_c < \gamma h$?
(Probabilistic analysis)

Tests:-

same lithological unit

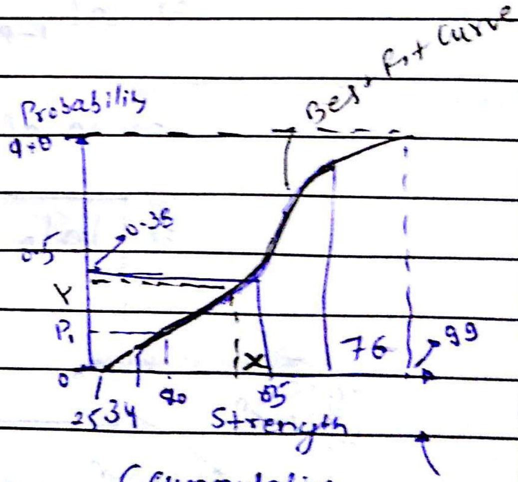
Values:-

50, 52, 34, 76, 74, 65, 68, 56, 63 (in MPa)

Probability that strength is less than or equal to γh. let $\gamma h = 55$ MPa

Graphical Method

(m)	Order	Probability
1	→ 34	0.1
2	50	0.2
3	52	0.3
4	56	0.4
5	63	0.5
6	65	0.6
7	68	0.7
8	74	0.8
9	→ 76	0.9



$$\text{Probability} = \frac{m}{n+1} = \frac{\text{order}}{\text{total values}+1}$$

Y: Probability of strength to be equal to or less than X,

$$\boxed{P(X \le x) = Y}$$

from this curve

$P(X \le 20) = 0$

$P(X \le 40) = P_1$

For failure $\boxed{P(X \le 55) = 0.38}$ → Probability of failure

Best equation for strength Data:-

Weibull distribution

Remember this exam

$$\boxed{P = 1 - e^{-\left(\frac{\sigma - \sigma_u}{\sigma_0}\right)^m}}$$

σ = variable

σ_u = minimum value of variable below which Probability of occurrence of the event is zero.

σ_0 = Scale Parameter = It determines the magnification of the plot

m = Steepness of the Plot

To fit into data:-

$$1 - P = e^{-\left(\frac{\sigma - \sigma_u}{\sigma_0}\right)^m}$$

$$\Rightarrow \frac{1}{1-P} = e^{\left(\frac{\sigma - \sigma_u}{\sigma_0}\right)^m}$$

$$\Rightarrow \ln\left(\frac{1}{1-P}\right) = \left(\frac{\sigma - \sigma_u}{\sigma_0}\right)^m$$

$$\Rightarrow \ln\ln\left(\frac{1}{1-P}\right) = m\ln\left(\frac{\sigma - \sigma_u}{\sigma_0}\right)$$

$$\Rightarrow \boxed{\ln\ln\left(\frac{1}{1-P}\right) = m\ln(\sigma - \sigma_u) - m\ln\sigma_0}$$

$$\boxed{Y = AX + B}$$

three unknown

two unknown

{ * least Square Method for minimizing the error
* Best fitting method }

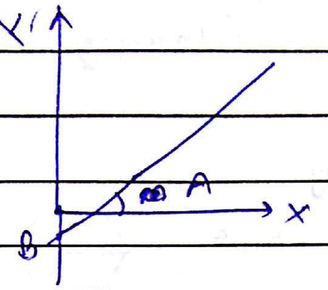
Steps

① assume any value of σ_u.

② Calculate X and Y.

Date : ___/___/___

m	Strength	Y	X



* Always (B will comes to be -ve)

So, $A = m$, $B = -A \ln \sigma_0$

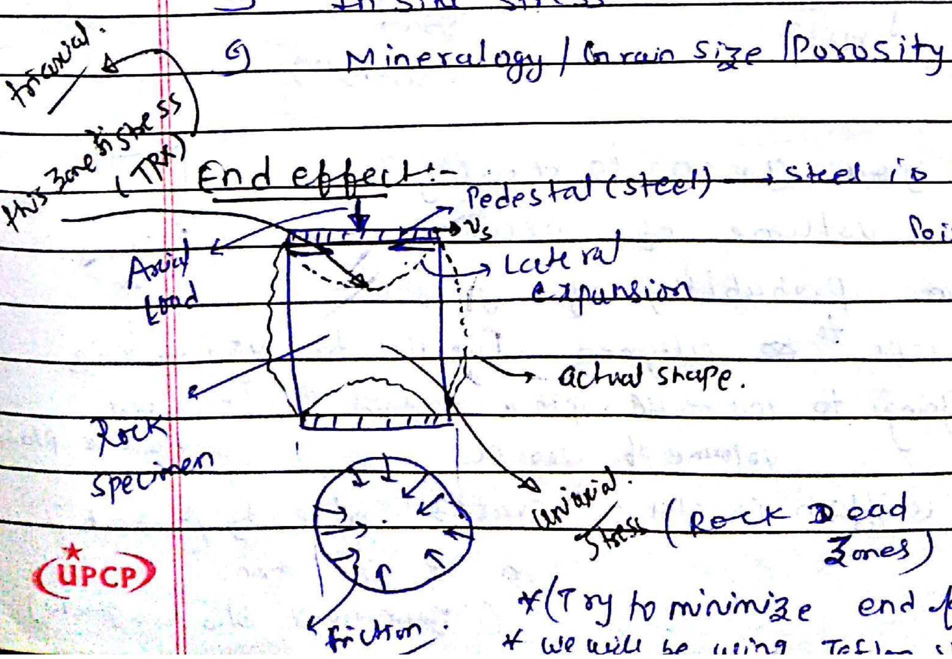
$\boxed{m \checkmark}$ $\boxed{\ln \sigma_0 = -\frac{B}{A}}$ $\sigma_0 \checkmark$

Factors Affecting UCS:-

UCS is a index Property. It is not a constant quantity. It depends on so many factors. These factors are :-

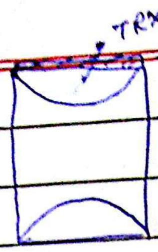
1) End effect
2) Specimen Geometry
3) Rate of Loading
4) Environmental Factor
5) In situ stress
6) Mineralogy / Grain size / Porosity

End effect:-



*(Try to minimize end friction)
* we will be using Teflon sheet (low φ value)

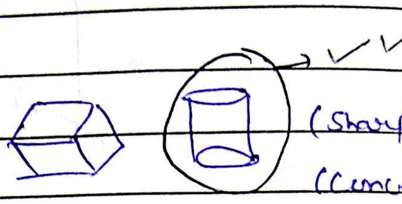
Date: TRX zone → low friction वाली use की जाती है |



For minimizing the end friction.
(Teflon sheet)

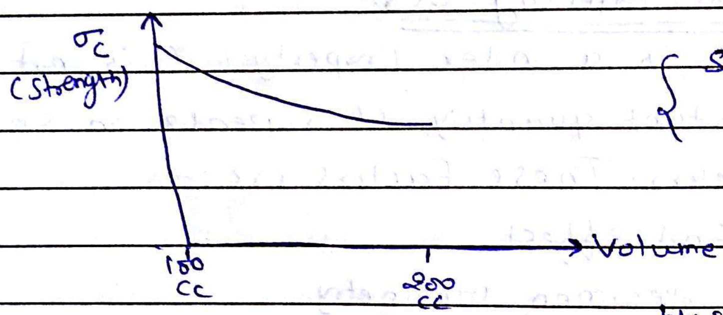
* or we can use spherical sheet
(For Reducing the eccentricity)
(loading will be in axial direction)

2) Specimen Geometry
→ depends on shape



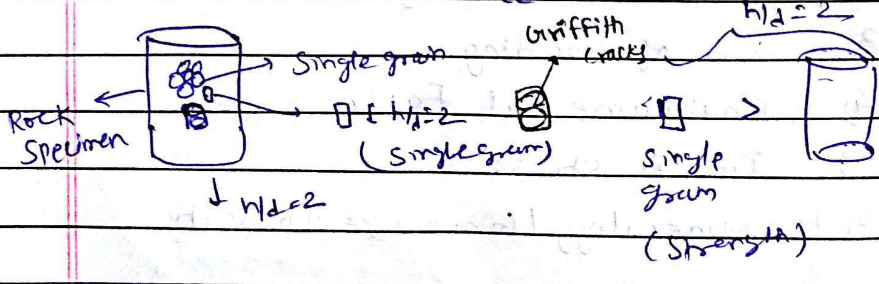
(Sharp corners)
(Concentration of Stress, premature failure)

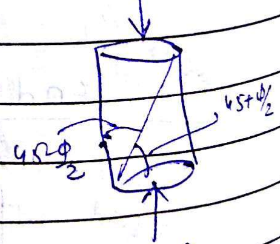
→ depends on size
→ $\frac{h}{d}$ ratio ↓ $\frac{h}{d} = ?$



{ Strength decrease
increase volume

Remember exam



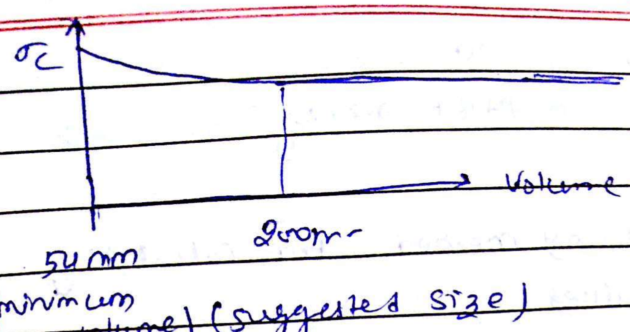


* As you go on increasing the volume of specimen than probability of griffith cracks to ~~be~~ aligned Parallel to 45+φ/2 is going to increase upto a certain volume of 200 CC.

45+φ/2 — potential failure plane

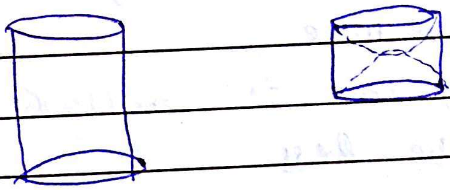
* Griffith Cracks :- Cracks already present in the specimen.
(Boundary b/w two grains)

Date : __/__/__



$\frac{h}{d}$ ratio

$\frac{h}{d}$ = large



$\frac{h}{d}$ = small

$\boxed{\frac{h}{d} \approx 2}$ → is used

$\boxed{\frac{h}{d} \approx 2 \text{ to } 2.5}$

if $\frac{h}{d} \neq 2$, than correct the values

Remember exam — Correction for $\frac{h}{d}$ values if it is not equal to 2.

① ASTM ⇒

$$\sigma_{c_1} = \frac{\sigma_c}{0.778 + \frac{0.222}{(h/d)}}$$

σ_{c_1} ∀ $\frac{h}{d} = 1$

σ_c ∀ $\frac{h}{d} \neq 1$

② Protodyakonov

$$\sigma_{c_2} = \frac{8\sigma_c}{7 + \frac{2}{(h/d)}}$$

→ ∀ $\frac{h}{d} = 2$

σ_c ∀ $\frac{h}{d} \neq 2$

∀ example

If $h = 90$ mm, $d = 50$ mm

and $\sigma_c = 50$ MPa

$\frac{h}{d} = \frac{90}{50} \Rightarrow 9/5$

So, corrected UCS values for $\frac{h}{d} = 2$

$$\sigma_{c_2} = \frac{8 \times 50}{7 + 2 \times 5/9} \Rightarrow \frac{200 \times 9}{73} \cong 25 \text{ MPa}$$

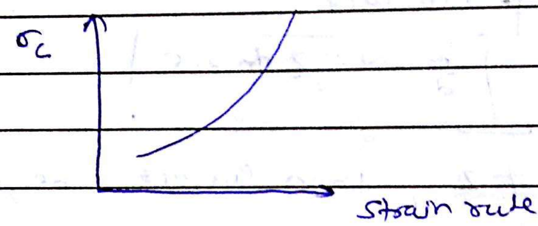
$\sigma_{c_1} \Rightarrow \dfrac{50}{0.778 + \dfrac{0.222 \times 5}{5}}$

$\forall \frac{h}{d} \geq 1$

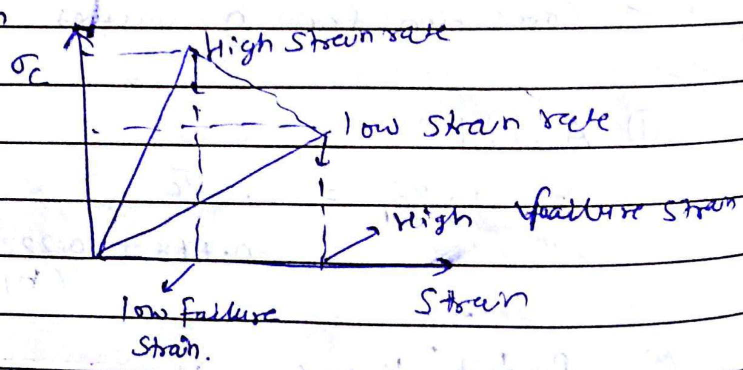
we can use any method for calculating corrected values.

3) Rate of Loading

* If you apply Higher loading rate, Strength will be more
* loading rate is small than Strength will be ~~less~~ less.



Failure Strain



* Strain Rate
  specimen should be such that failure occurs within about 5 to 10 minutes

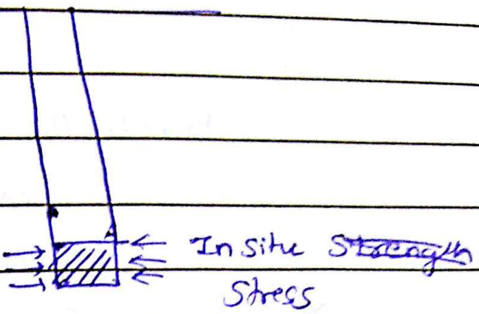
4) Environmental factors:- → Moisture Content → Type of Liquid → Temperature

* Moisture Content may result in low strength
* Type of liquid — chemicals.
* at large depth — Temperature increases.

- On High temp. low strength

5) In-situ Stress :-

* Removal of stress occurs when you take out from the ground



* Induce cracks and result in lower strength.

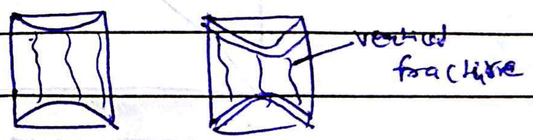
6) Mineralogy :-
Porosity, grain size
well graded Material => Higher strength
* Strength will depend on minerals, grain size distribution, Porosity etc.

UCS :-

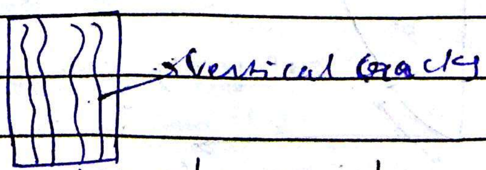
Factors affecting UCS :-

** Mode of failure :-
(brittle / ductile) → Type of material

(i) Cone Formation
(ii) constrained shear due to end-friction.

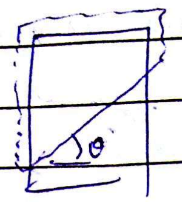


(ii) If there is no end-friction and material is brittle



It is called splitting / slabbing / vertical splintering

(ii) Shear Failure

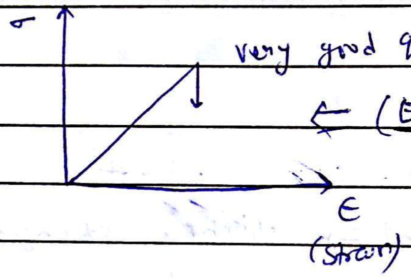


$\theta = 45 + \phi/2$

Practically => one mode is dominating, others may also be present

※ Shape of stress strain curve

(1) (Axial stress)

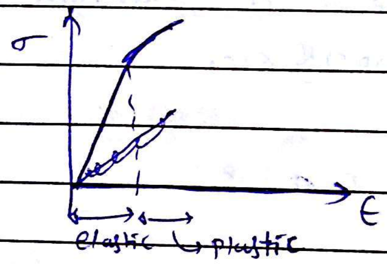


very good quality rock

← (Elastic)

Quartzite
(elastic - पूरा का पूरा)

(2)

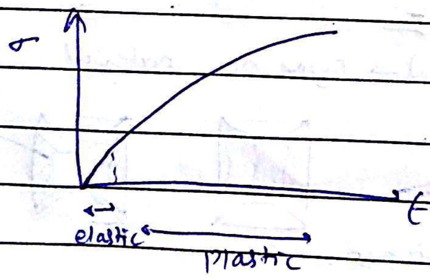


⇐ Elastoplastic

Basalt

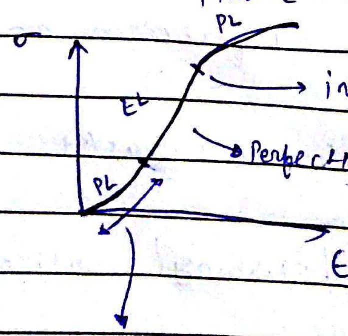
(elastic - large
plastic - small)

(3)



ex Sandstone
Elasto-Plastic
elastic - less
plastic - large

(4) Most common shape ✱



ex :- Marble

→ initiation of cracks

→ Perfectly elastic Behaviour (cracks are closed)

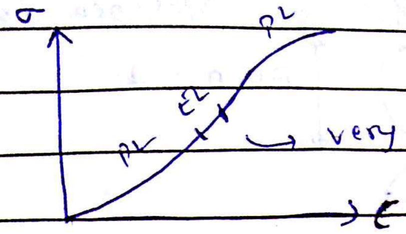
PL - EL - PL

✱ Initial seating (adjustment)

✱ Closure of microcracks.

Date : __/__/__

(5)



→ very small elastic Behaviour.

PL-EL-PL

~~for ex:-~~ Schist

(6)

EL-PL-Creep

Indirect Methods of UCS:-

In Field Symmetrical specimens are not always available.

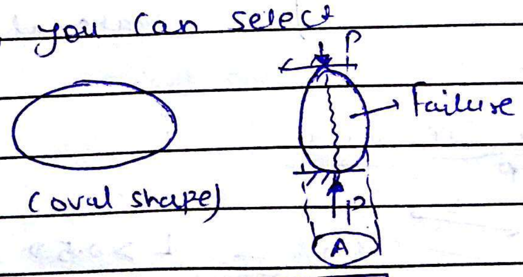
Basic Principle is

energy required to split the Particle that will give you idea about the Strength.

For that Purpose there is a test:

(A) ISRM (International Standards for Rock Mechanics)

So, you can select

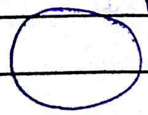


Remember $\boxed{\sigma_c \simeq \frac{1}{0.19} \frac{P}{A}}$

A = Area Projected on Horizontal plane

(B) Point Load Test (Common)

Rounded Regular specimens



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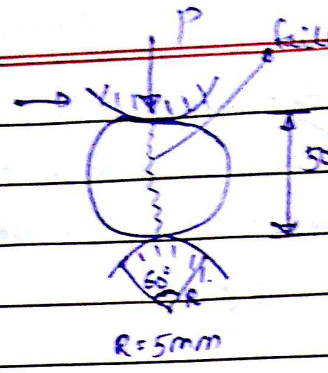


Standard Sized Cone → Made up of Hardend Steel → used to compress Rounded specimen

Specimen is loaded

Point Load Strength Index

Remember exam

$$\boxed{I_s = \frac{P}{d^2}}$$

Standard value of $d = 50$ mm

So, I_{s50}

$$\boxed{I_{s50} = \frac{P}{(50)^2}}$$

d = distance b/w Cones.

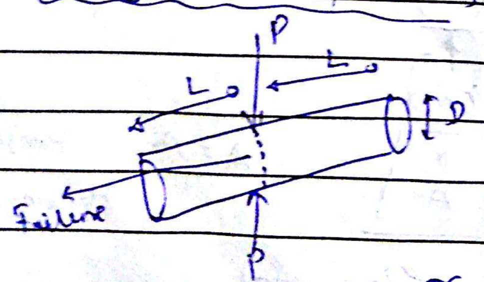
$$\boxed{\sigma_c \simeq 24\, I_{s50}}$$

* what if $d \neq 50$, than you have to apply the Corrections.

Correction Factor $(F) = \left\{\frac{d}{50}\right\}^{0.45}$ — 0.45 = Based on experience (general value)

So, $\boxed{I_{s50} = F\left[\frac{P}{d^2}\right]}$ Applicable for Isotropic Material

For Cylindrical Specimens

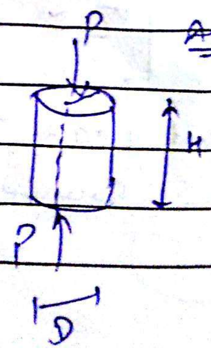


$L > 0.5D$

Dia Metral

or,

Axial



$H = \left(\frac{1}{3} \text{ to } 1\right) D$

Date: ___/___/___

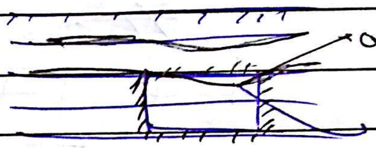
Above Tests are not used in for Projects. These are just index test. For Project we need to do actual UCS test.

Tensile strength :-

Field — soil → $\boxed{\text{Tensile strength of soil} \approx 0}$

but Rocks do have some tensile strength. but not always.

For example :-



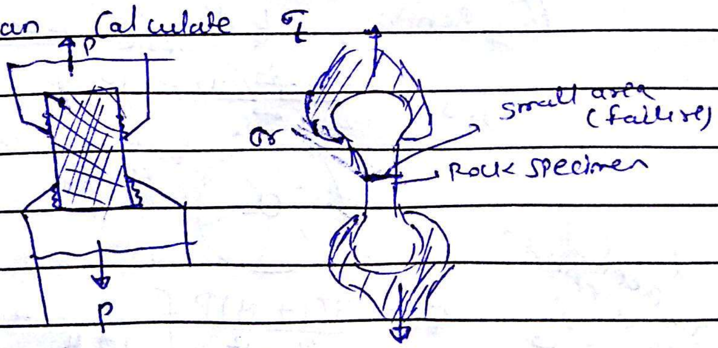
So, in some situation Tension strength important हो जाती है

test — direct / indirect

direct test :- Tensile stress is induced and specimen is going to fail due to tensile stress.

So, we can calculate σ_t

Problem is



From practice point of view, these tests are not common.

indirect tests :- (Theory of elasticity)

(1) Bending test

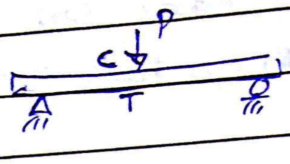
(2) Hydraulic Extension

(3) Diametral compression

Date : ____/____/____

(4) Miscellaneous

→ bending test

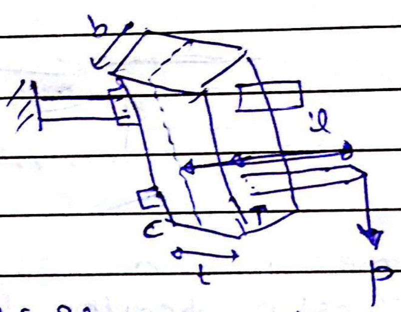


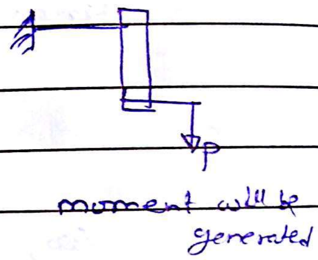
Max. Bending Moment $= \dfrac{PL}{4} = M$

$$\boxed{\sigma_t = \frac{6M}{bd^2}}$$

→ ~~Hydraulic extension~~

→ Protodyakonov



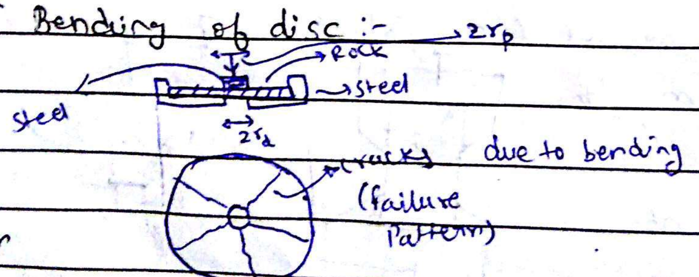


$M = Pl$ = moment

$$\sigma = \frac{My}{I} + \frac{P}{A}$$

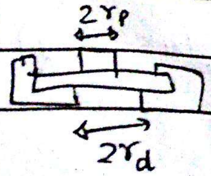
$$\boxed{\sigma = \frac{My}{I} + \frac{P}{A}}$$

Bending of disc :-



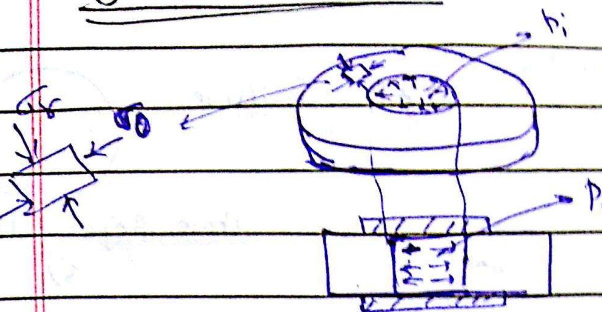
* no need to remember sir will give the this in exam

$$\sigma_t = \frac{3(1+\mu)P}{2\pi t^2}\left[\frac{1}{1+\mu} + \log_e \frac{r_d}{r_p} - \left(\frac{1-\mu}{1+\mu}\right)\frac{r_p^2}{4r_d^2}\right]$$



Date : / /

Hydraulic extension



assumption :- Failure is due to tension

Remember

$$\sigma_\theta = -p_i\left(\frac{r_i^2}{r_o^2 - r_i^2}\right)\left[\frac{r_o^2}{r^2} + 1\right]$$

−ve means Tensile

r_i = internal Radius

r_o = outer Radius

r = distance from center

$$\sigma_r = p_i\left(\frac{r_i^2}{r_o^2 - r_i^2}\right)\left(\frac{r_o^2}{r^2} - 1\right)$$

at $r = r_i$ (failure will occur) (Maximum tensile ~~force~~ Stress)

$$\text{so, } (\sigma_\theta)_{\text{at } r = r_i} = -p_i\left(\frac{r_i^2}{r_o^2 - r_i^2}\right) \cdot \left[\frac{r_o^2 + r_i^2}{r_i^2}\right]$$

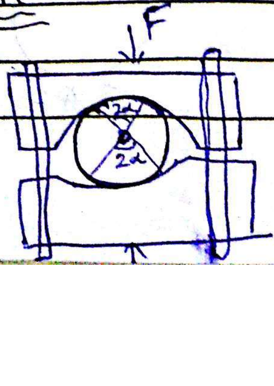
$$\boxed{\sigma_\theta = -p_i \frac{(r_o^2 + r_i^2)}{(r_o^2 - r_i^2)}} \text{ or } \sigma_t \quad \boxed{\text{tangential tensile stress}}$$

Above test is practical difficult

so, most common test in practical purpose is

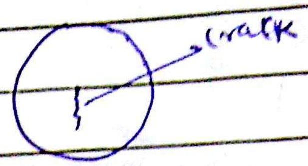
Most Popular

Brazilian Test :-



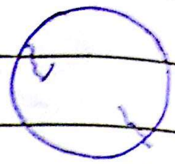
$\alpha = 11^\circ$
$2\alpha = 22^\circ$

Failure should initiate from center



($\checkmark$)

If failure



then Reject this test
($\times$)

Remember

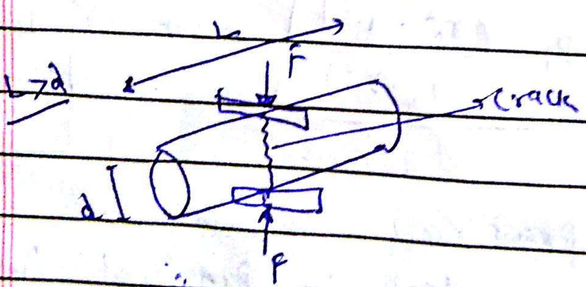
$$\boxed{\sigma_t = \frac{F}{\pi r_0 t}}$$

(54 mm)

* Wanted Minimum NX size of specimen is Preferred.
* Thickness 't' is kept between 0.5 to 1 times Diameter (D).
  If 't' is small than buckling will takes Places.

Miscellaneous Test :-

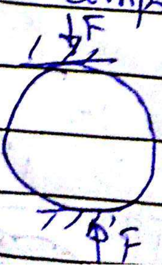
(a) Diameteral Compression of Cylinder



$$\boxed{\sigma_t = \frac{0.96\,F}{d^2}}$$

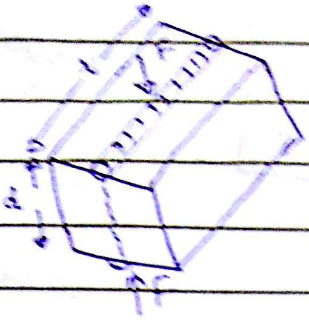
(b) Spehere diameteral Compression

$$\boxed{\sigma_t = \frac{0.90\,F}{d^2}}$$



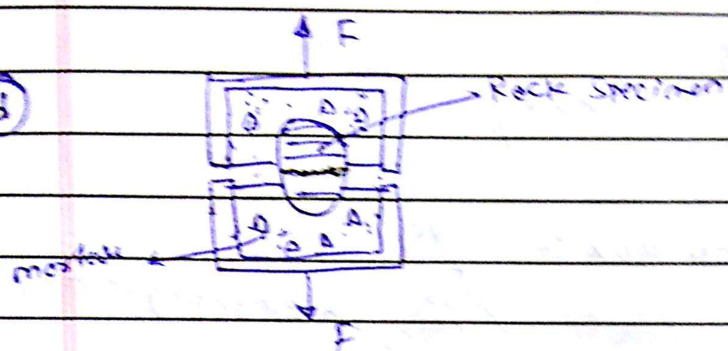
Date : ___/___/___

(c)



$$\boxed{\sigma_t = \frac{2F}{\pi d t}}$$

(d)



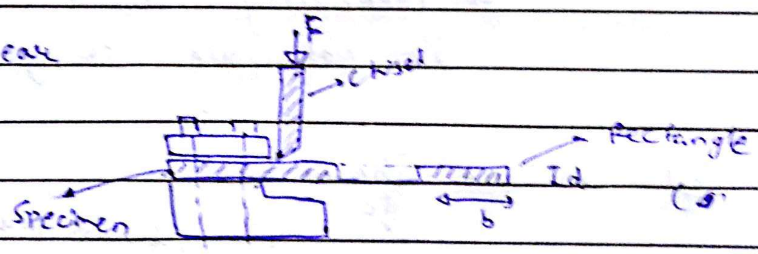
only when Rock is weak.

(e) Point load Test

$$\sigma_t \simeq 1.25\, I_{s50}$$

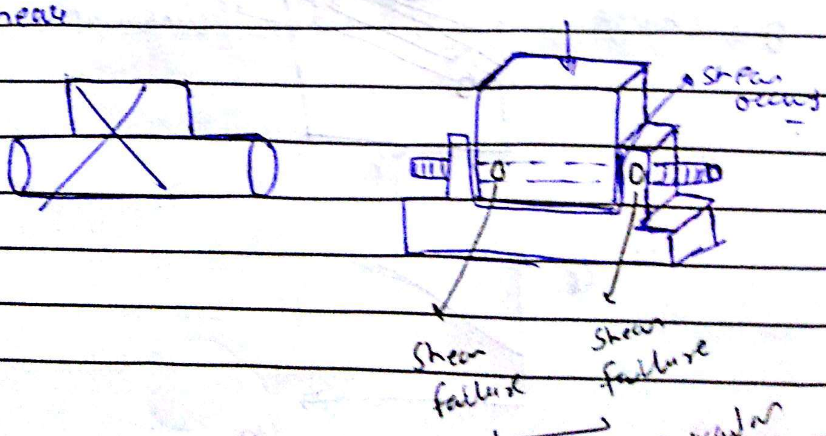
Shear Test :-

(1) Single Shear



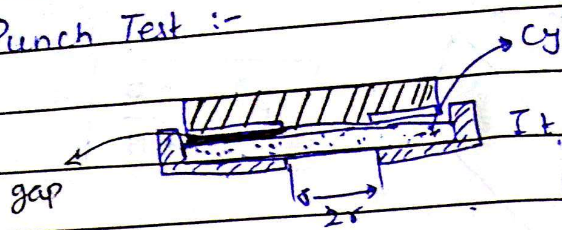
Shear area = bd

(2) Double shear



two circular shear failure

③ Punch Test :-



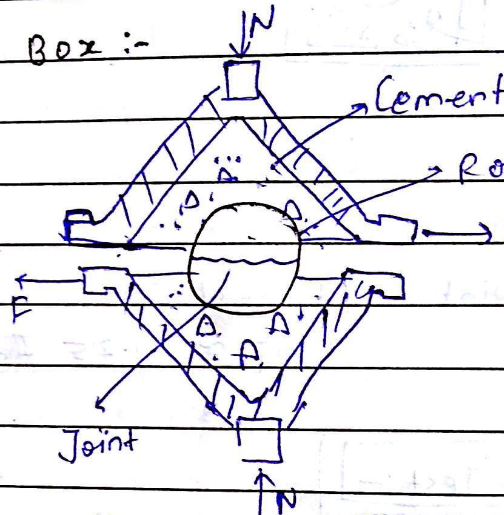
(no bending, Punching)

Shear area = $2\pi r t$

$$\boxed{\sigma_t = \frac{F}{2\pi r t}}$$

④ Field Shear Box :-

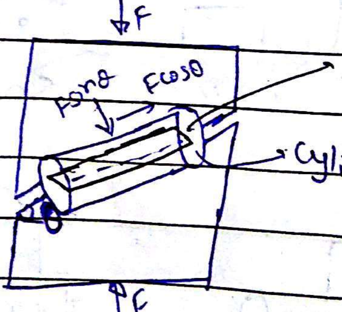
For very very weak Rock

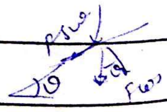


* Generally used for jointed rock. To calculate strength of joint.

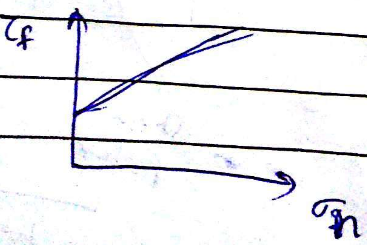
⑤ Oblique Shear Test.

θ is change

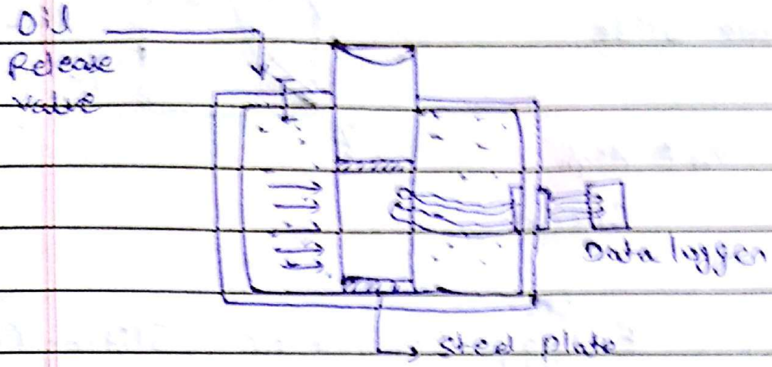




Shear area : DL



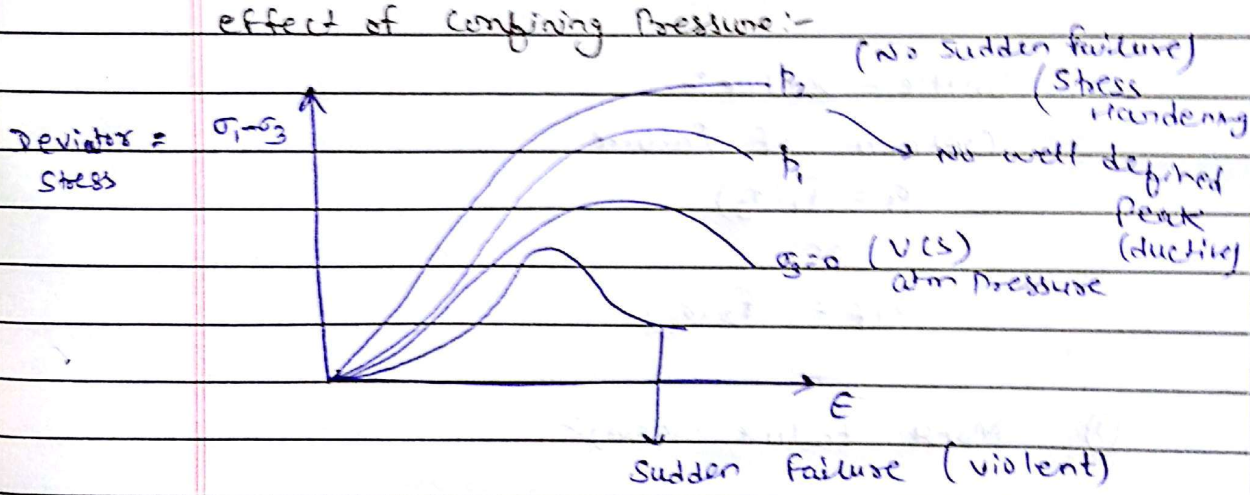
Tri-axial Behaviour of Rocks → Intact Rocks:-



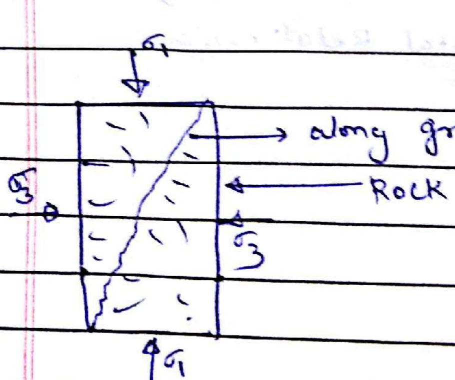
$\sigma_{1f} = f(\sigma_3)$

σ_{1f}	σ_3
50	1
70	2
65	3

effect of confining Pressure:-



Sudden failure of energy
(Brittle)



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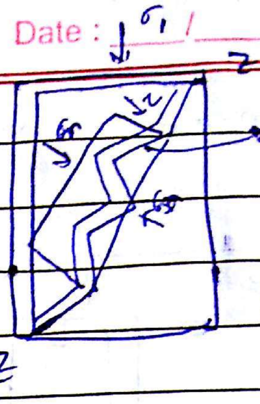


with Shear failure or Slide over (work don adjust σ_n)

(a) If σ_n = low

~~Sde~~ Slide

(b) If σ_n = High

Shear

Low σ_n :- Energy Required for Sliding (dilatancy) is low brittle failure.

High σ_n :- E(Shear failure) = Low

$\therefore$ Shear failure

Ductile failure.

Brittle – Ductile :-

Criteria of Failure

$\sigma_1 = f_1(\sigma_3)$

or

$\sigma_{1f} = f_2(\sigma_3)$

(1) Mohr failure theory :-

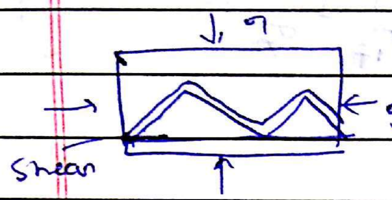
The Failure of a material may be Represented by a fundamental Relationship.

Tri-axial strength :-

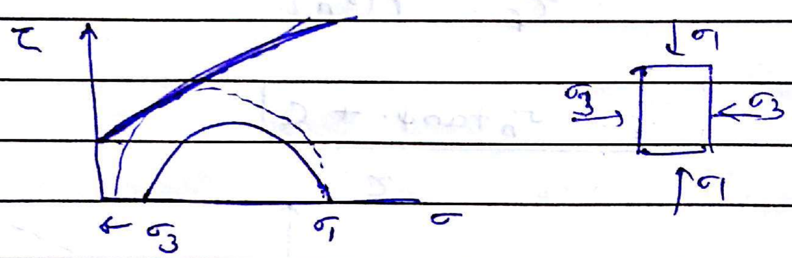
Failure Criteria :-

→ strength increases on increasing confining Pressure.

→ If σ_3 is high, at failure, change in volume for dense sand will decrease. It is not going to dilate.

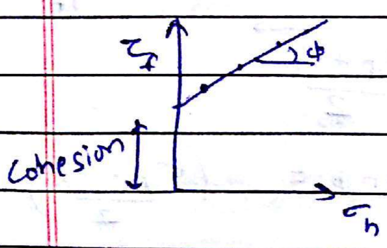


Mohr Theory :-



Coulomb Theory :-

"Direct Shear Test"

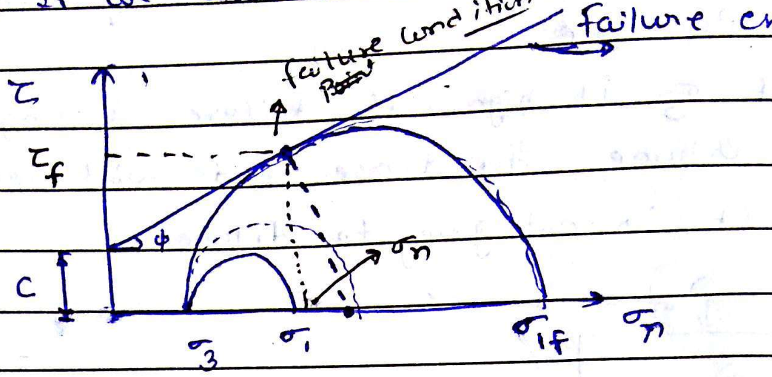


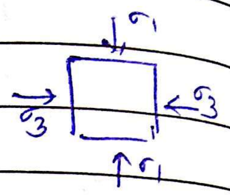
$\tau_f \Rightarrow$ Stress dependent, Stress independent } two components

$$\tau_f = \underbrace{\sigma_n \tan\phi}_{\text{stress dependent}} + \underbrace{c}_{\text{Cohesion: stress independent}}$$

Mohr - Coulomb Theory :-

If we assume linear failure Criteria



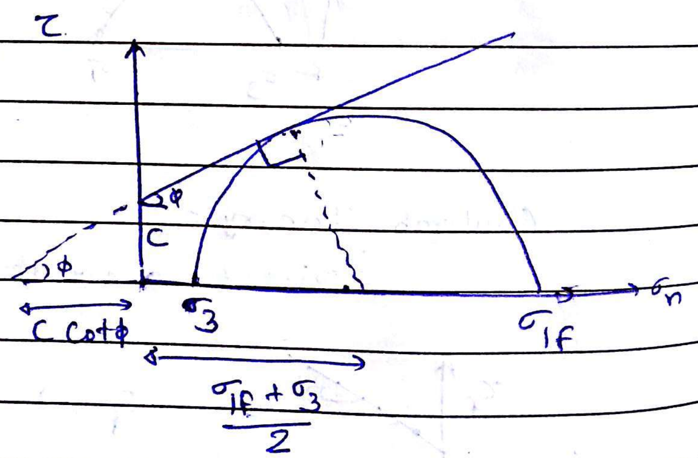


failure Criteria

$$\sigma_{1f} = f(\sigma_3)$$

$$\tau_f = f(\sigma_n)$$

$$\boxed{\tau_f = \sigma_n \tan\phi + C}$$



$$\left(c\cot\phi + \frac{\sigma_{1f}+\sigma_3}{2}\right)\sin\phi = \left(\frac{\sigma_{1f}-\sigma_3}{2}\right)$$

$$\Rightarrow c\cos\phi + \left(\frac{\sigma_{1f}+\sigma_3}{2}\right)\sin\phi = \left(\frac{\sigma_{1f}-\sigma_3}{2}\right)$$

$$\Rightarrow 2c\cos\phi + \sigma_{1f}[\sin\phi - 1] + \sigma_3[\sin\phi + 1] = 0$$

$$\sigma_{1f} = \frac{\sigma_3(1+\sin\phi) + 2c\cos\phi}{1-\sin\phi}$$

Date: / /

$\sigma_{1f} = f(\sigma_3)$

$$\boxed{\sigma_{1f} = \sigma_3 \tan^2(45 + \phi/2) + 2c\,\frac{\cos\phi}{1-\sin\phi}}$$

~~σ_{1f} σ_3 tan²~~

Mohr Coulomb criterion

$$\boxed{\sigma_{1f} = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\left(\frac{\cos\phi}{1-\sin\phi}\right)}$$

Example

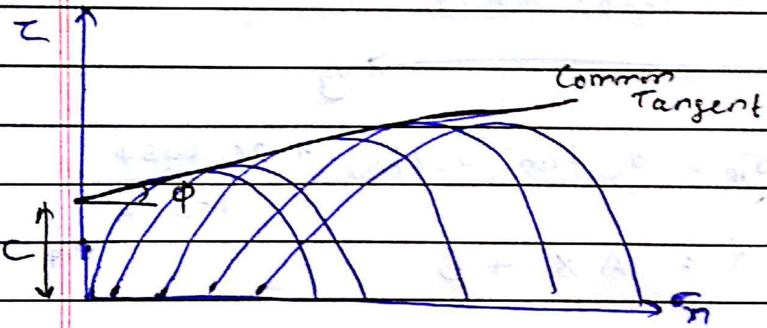
Tri-axial Test Result:-

Lab ⇒ $c, \phi = ?$

σ_3	σ_1
0	70
5	85
10	97
15	107
20	105

↓ at the ~~time~~ failure.

(i) Draw Mohr Circles



(Graphically)

It is very difficult to get best fitting common tangent.

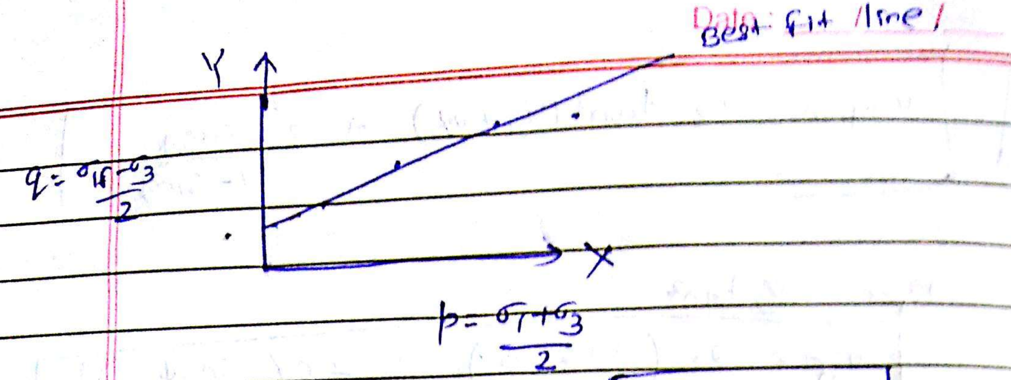
(ii) So, we plot p-q plot.

p-q plot

$$\frac{\sigma_{1f}-\sigma_3}{2} = c\cos\phi + \left(\frac{\sigma_{1f}+\sigma_3}{2}\right)\sin\phi$$

$$\boxed{Y = B + AX}$$ eqn of straight line.

UPCP

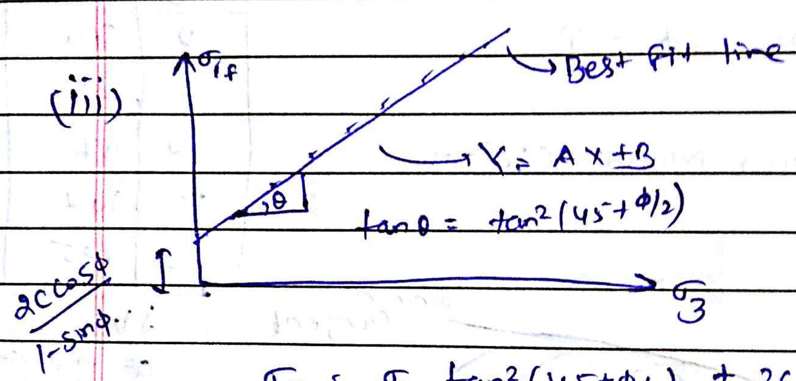


$A = \sin\phi$ $\quad$ $\boxed{\phi = \sin^{-1} A}$

$B = c\cos\phi$

$B = c\sqrt{1-A^2}$ $\Rightarrow$ $\boxed{c = \frac{B}{\sqrt{1-A^2}}}$

(iii)

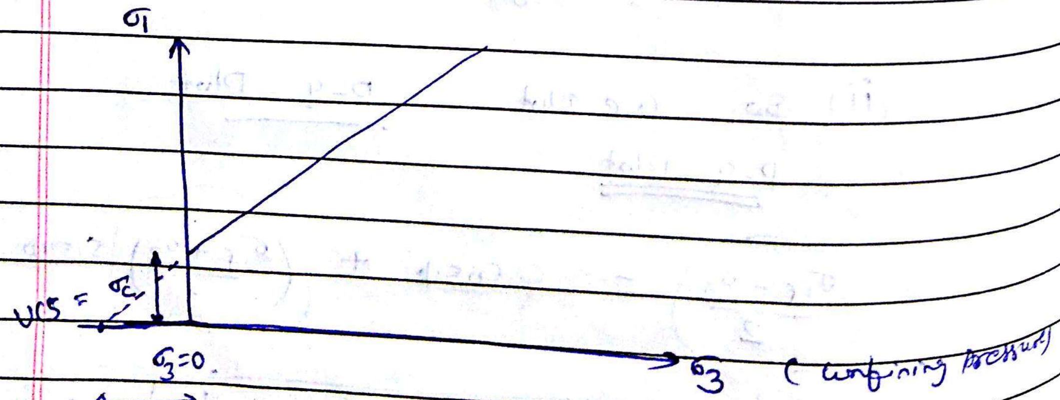


$$\sigma_{1f} = \sigma_3 \tan^2(45+\phi/2) + \frac{2c\cos\phi}{1-\sin\phi}$$

$$Y = AX + B$$

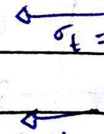
$$\tan^2(45+\phi/2) = A$$

$$\frac{2c\cos\phi}{1-\sin\phi} = B$$



σ_t = tensile strength (fail at $\sigma_1 = 0$)

(failure due to tension)



Pull

$$\boxed{\frac{\sigma_c}{\sigma_t} = \tan^2(45 + \phi/2)}$$

$$\sigma_1 = \sigma_3 \frac{1+\sin\phi}{1-\sin\phi} + \frac{2C\cos\phi}{1-\sin\phi}$$

$$\Rightarrow \sigma_c = 0\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + \frac{2C\cos\phi}{1-\sin\phi}$$

$$\boxed{\sigma_c = \frac{2C\cos\phi}{1-\sin\phi}}$$

$$0 = (-\sigma_t)\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + \frac{2C\cos\phi}{1-\sin\phi}$$

$$\boxed{\sigma_t = \frac{2C\cos\phi}{1+\sin\phi}}$$

$$\boxed{\frac{\sigma_c}{\sigma_t} = \frac{1+\sin\phi}{1-\sin\phi} = \tan^2(45 + \phi/2)}$$

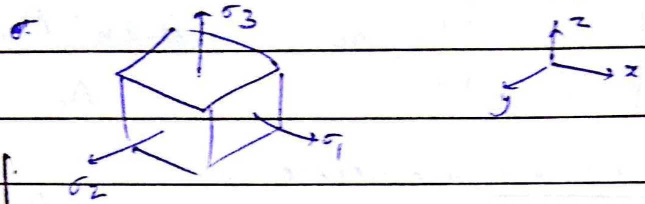
$\boxed{\sigma_c, \sigma_t, C, \phi}$ → four things (rest other things we can find)

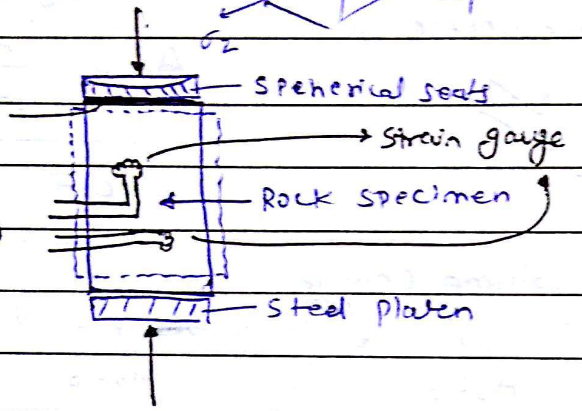
(use)

Lab Tests:-

1) UCS (Uniaxial compression Test)
↓
(uniaxial compressive strength Test)

In Rocks all three principle stress are acting.

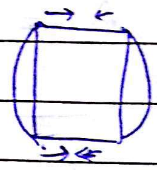




↓

(To minimize the end friction)

. If end friction is not minimize than deflected shape will Bending will be there



So, vertical stress will not be more a principal stress. because due to friction shear will develope.

* Strain gauge is used to measure the strain at particular point. [Costly]

$\boxed{\varepsilon = \frac{\Delta L}{L}}$ (or we can use dial gauge) → strain is uniform throughout the length (assumption)

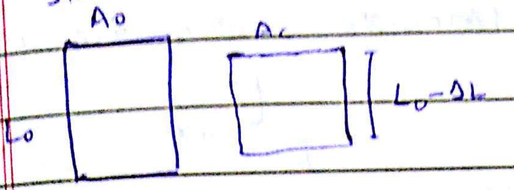
Load → strain controlled (Rate of deformation is specified) → Most labs have this type of arrangement
Load → stress controlled (5 MPa/minute) ~~(most Realistic)~~ → (very costly) (computer controlled)

Load

Deformation

Stress (Area correction [illegible])

Strain



$\Rightarrow A_0 L_0 = A_c (L_0 - \Delta L)$

$$A_c = \frac{A_0 L_0}{L_0 - \Delta L}$$

$$A_c = \frac{A_0}{1 - \Delta L / L_0}$$

$$\boxed{A_c = \frac{A_0}{1 - \epsilon}}$$

assumption :- No volume change

$$\boxed{A_c = \frac{A_0}{1 - \epsilon}}$$ ✓

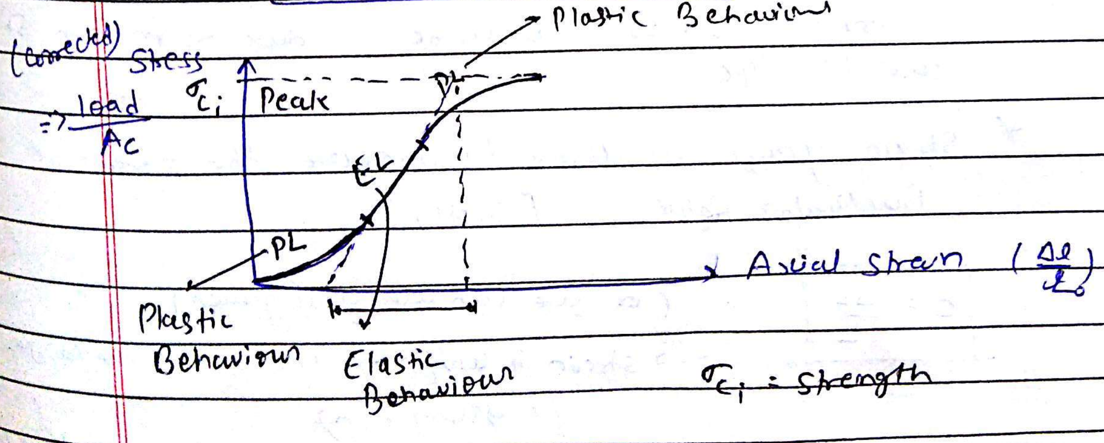
If there is a volume change

$$\epsilon_v = \frac{\Delta V}{V_0} = \frac{\Delta V}{A_0 L_0} = \frac{(\text{Final volume} - \text{initial vol.})}{A_0 L_0}$$

$$\boxed{A_0 L_0 + \epsilon_v A_0 L_0 = A_c (L_0 - \Delta L)}$$

$$A_c = \frac{A_0 + \epsilon_v A_0}{1 - \epsilon} \Rightarrow \frac{A_0 (1 + \epsilon_v)}{(1 - \epsilon)}$$

$$\boxed{A_c = \frac{A_0 (1 + \epsilon_v)}{1 - \epsilon}}$$

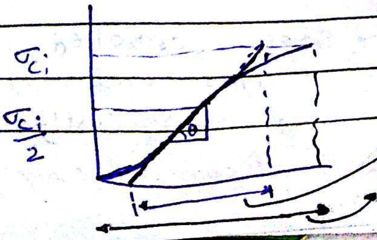


σ_{c_i} = Strength

Modulus :-

tangent Modulus

$\Rightarrow$ slope of curve at any point



ϵ_e = elastic strain

ϵ_f = failure strain

$\tau = \sigma_n \tan\phi + c$

ϕ = Increase in shear ~~strength~~ stress per unit increase in normal stress

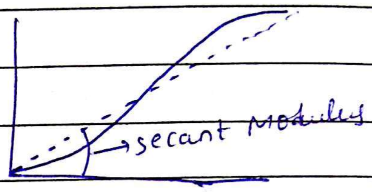
Date : ___/___/___

tangent Modulus = slope of curve at any value

$$E_{\epsilon 50} = \frac{\sigma_{ci}}{\epsilon_e} \quad \text{(at 50\% Peak stress)}$$

initial Tangent Modulus :- $E_{\epsilon 0}$

Secant Modulus = E_{sec}



Get :- σ_{ci} = Strength

$E_{\epsilon 50}$ = Tangent Modulus

$$\text{Modulus Ratio} = \frac{E_{\epsilon 50}}{\sigma_{ci}} \Rightarrow \frac{1}{\epsilon_e}$$

It gives the idea about elastic strain at failure.

* It gives the idea about elastic strain at time of failure and gives some idea about failure strain (ϵ_f) dependending on the shape of curve. It is lower limit of failure strain. $\boxed{\epsilon_f \geq \epsilon_e}$

Classify the Rocks :-

Based on "strength" & "Modulus Ratio".

(This is called Deere-Miller classification)

(Two lettered)

Remember This Table

Strength			Modulus Ratio	
A -	Very High	> 200 MPa	H - High	> 500
B -	High	100-200 MPa	M - Medium	200-500
C -	Medium	50-100 MPa	L - Low	< 200
D -	Low	25-50 MPa		
E -	very low	< 25 MPa		

Strength → XY ← Modulus Ratio

(UPCP)

Date : ___/___/___

Eg. Rock which is classify as DL

low Strength and Low modulus Ratio

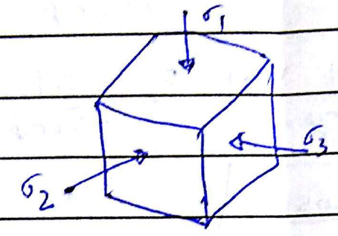
CM => Medium , Medium
(50 to 100) (200-500) → $\underline{\underline{\epsilon_e = \left(\frac{1}{500} \text{ to } \frac{1}{200}\right)}}$

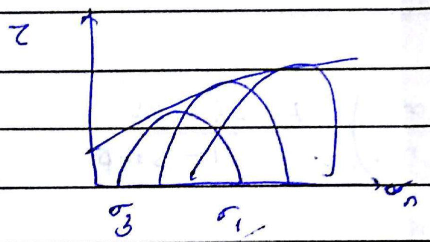
28/2/14

After Mid Term

Failure Criteria for Intact Rocks:-

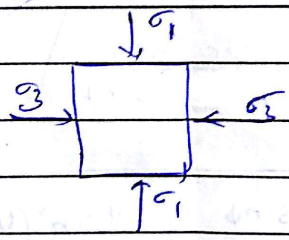
Mohr Theory — already discussed



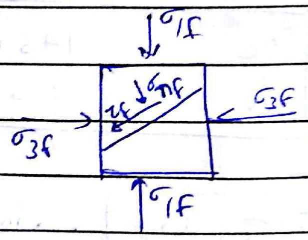


no effect of σ_2

So,

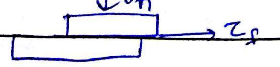


at time of failure



Coulomb Theory

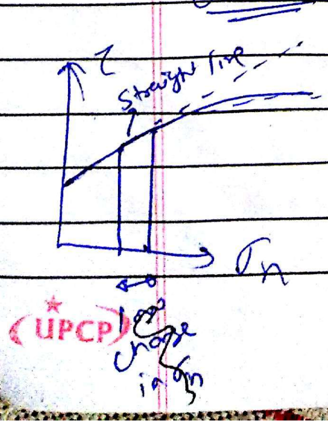
(at failure)

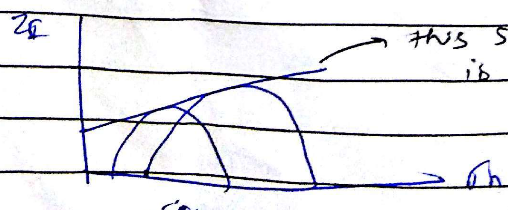


$\tau_f = \sigma_n \tan\phi + c$

(coulomb's eqn)

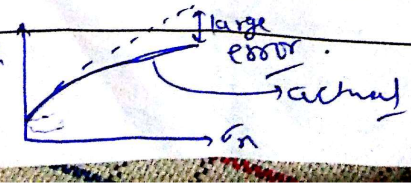
Combined (Mohr Coulomb failure ~~Theory~~) Criteria)





this straight line is valid only for small change (~~stress~~) condition

small stress condition



1

Date : ___/___/___

shear strength → τ_f ; stress dependent → $\sigma_n \tan\phi$; stress independent → c

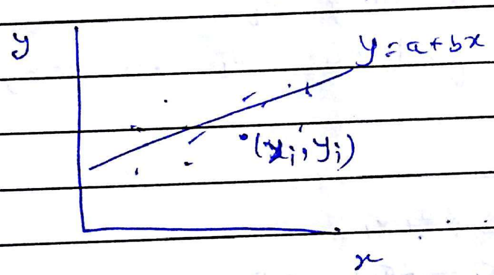
$$\tau_f = \sigma_n \tan\phi + c$$

$$\sigma_{1f} = \left(\frac{2c\cos\phi}{1-\sin\phi}\right) + \sigma_{3f}\left(\frac{1+\sin\phi}{1-\sin\phi}\right)$$

Parameters :- $c, \phi \rightarrow$ shear strength (stress dependent)

shear strength (stress independent)

Least square method for optimal Result.
(Straight line fitting)



$y = a + bx$

$y_i = a + bx_i$

$(a + bx_i - y_i) = \text{error}$

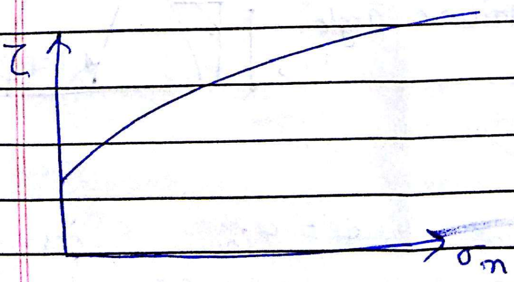
minimize the error (square).

$y = a + bx$

$\Sigma y = \Sigma a + b\Sigma x \longrightarrow (1)$

$\Sigma xy = a\Sigma x + b\Sigma x^2 \longrightarrow (2)$

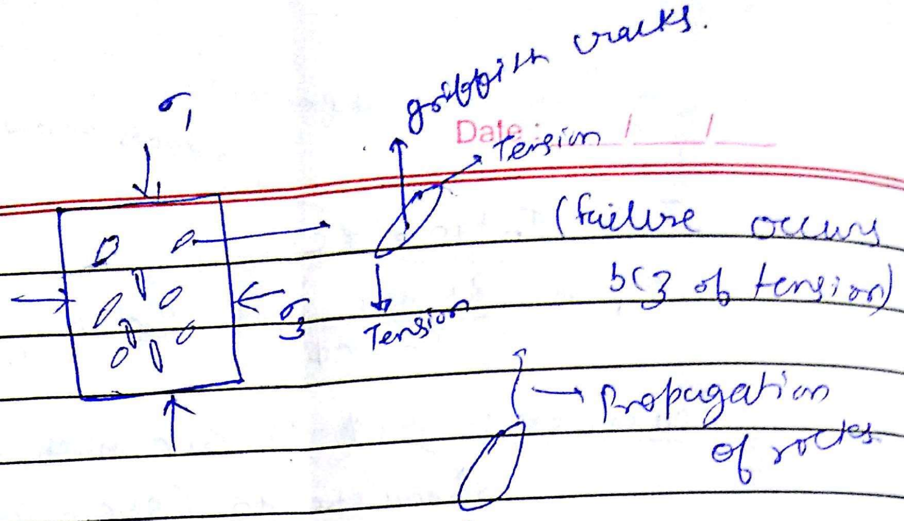
~~b=~~ ~~$N\Sigma x$~~



Non linear failure Criteria :-

(1) Griffith criteria
for Brittle Material – "glass"

UPCP



exam

$$\boxed{(\sigma_1 - \sigma_3)^2 = 8\sigma_t(\sigma_1 + \sigma_3)} \quad \text{for } \sigma_1 + 3\sigma_3 > 0$$

Parameter is only 'σ_t'.

You can find out σ_1 for any σ_3

For $\sigma_3 = 0$

$\Rightarrow \quad \sigma_1^2 = 8\sigma_t\sigma_1$

$\sigma_1(\sigma_1 - 8\sigma_t) = 0$

$\boxed{\sigma_c =} \boxed{\sigma_1 = 8\sigma_t}$ → UCS value when $\sigma_3 = 0$.

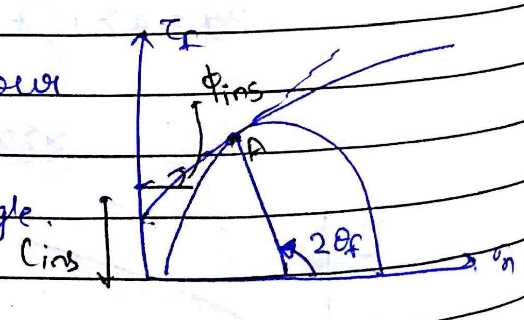
$$\boxed{\frac{\sigma_c}{\sigma_t} = 8}$$

Non linear Behaviour

If non linear behaviour

exam

θ_f = failure angle.



for Plane A,

$C = C_{ins}$, $\phi = \phi_{ins}$ ins = instantaneous

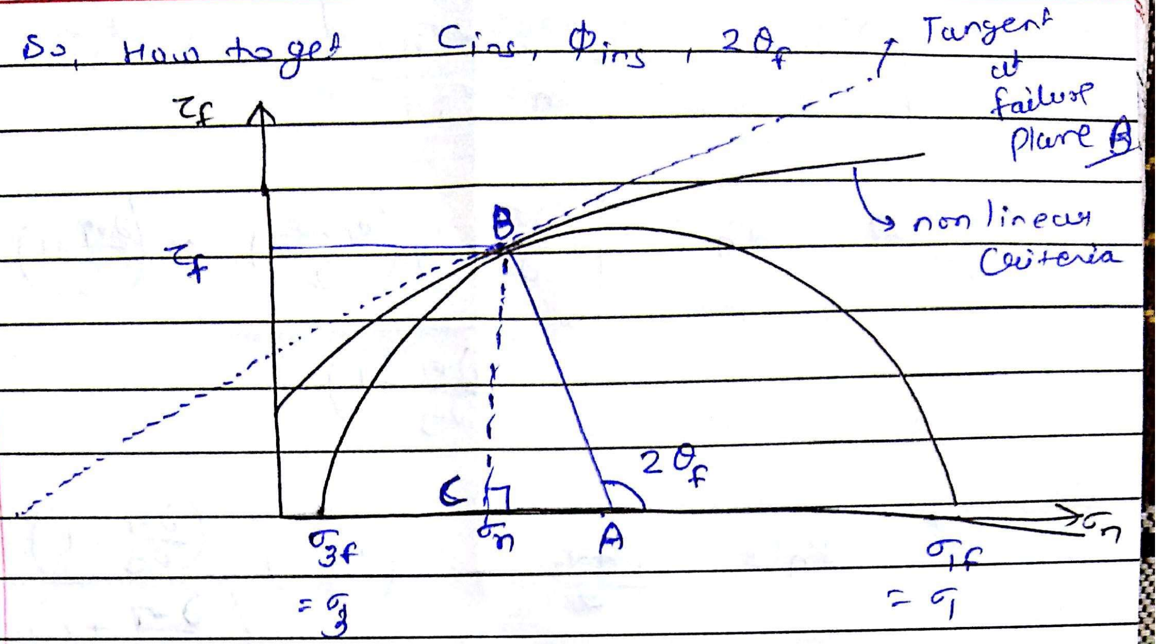
corresponding to $2\theta_f$

actually in practicle c and ϕ is not constant, because of non linear behaviour

Date : ___/___/___

exam

So, How to get c_{ins}, ϕ_{ins}, $2\theta_f$



(at time of failure)

Balmer expressions:

exam

θ_f = angle of failure plane w.r.t Horizontal

$$\sigma_{1f} = f(\sigma_{3f}) \longrightarrow (1)$$

From ΔABC

$$\Rightarrow \left(\frac{\sigma_{1f} - \sigma_{3f}}{2}\right)^2 = (\tau_f)^2 + \left(\frac{\sigma_{1f} + \sigma_{3f}}{2} - \sigma_n\right)^2 \longrightarrow (1)$$

Partially differentiate above eqn w.r.t σ_{3f}

$$\Rightarrow \cancel{2\left(\frac{\sigma_{1f} - \sigma_{3f}}{2}\right)\left[\frac{1}{2}\right] = 0}$$

$$\Rightarrow 2\left[\frac{\sigma_1 - \sigma_3}{2}\right]\left[\frac{\partial \sigma_1}{\partial \sigma_3} - 1\right]\frac{1}{2} = 0 + 2\left[\frac{\sigma_1 + \sigma_3}{2} - \sigma_n\right]\left[\frac{\partial \sigma_1}{\partial \sigma_3} + 1\right]\frac{1}{2}$$

$\Rightarrow \sigma_n$

$$\Rightarrow \left(\frac{\sigma_1 - \sigma_3}{2}\right)\left(\frac{\partial \sigma_1}{\partial \sigma_3} - 1\right) = 0 + \left[\frac{\sigma_1 + \sigma_3}{2} - \sigma_n\right]\left[\frac{\partial \sigma_1}{\partial \sigma_3} + 1\right]$$

instantaneous

$$\frac{\frac{\partial \sigma_1}{\partial \sigma_3} - 1}{\frac{\partial \sigma_1}{\partial \sigma_3} + 1} = \frac{\frac{\sigma_1 + \sigma_3}{2} - \sigma_n}{\frac{\sigma_1 - \sigma_3}{2}}$$

$$\Rightarrow \frac{\partial \sigma_1}{\partial \sigma_3} = \frac{\sigma_1 + \sigma_3 - 2\sigma_n}{\sigma_1 - \sigma_3 - \sigma_1 - \sigma_3 + 2\sigma_n} \Rightarrow \frac{\sigma_1 + \sigma_3 - 2\sigma_n}{2\sigma_n - 2\sigma_3} \quad (2)$$

Date : ___/___/___

$$\boxed{\frac{\partial\sigma_1}{\partial\sigma_3} = \frac{\sigma_1+\sigma_3-2\sigma_n}{\sigma_n-\sigma_3}}$$

$$\Rightarrow \quad \sigma_n = -\frac{\left[\left(\frac{\partial\sigma_1}{\partial\sigma_3}-1\right)\left(\frac{\sigma_1-\sigma_3}{2}\right) - \left(\frac{\partial\sigma_1}{\partial\sigma_3}+1\right)\left(\frac{\sigma_1+\sigma_3}{2}\right)\right]}{\left(\frac{\partial\sigma_1}{\partial\sigma_3}+1\right)}$$

$$\sigma_n = \frac{\sigma_1+\sigma_3}{2} - \left(\frac{\sigma_1-\sigma_3}{2}\right)\frac{\left(\frac{\partial\sigma_1}{\partial\sigma_3}-1\right)}{\left(\frac{\partial\sigma_1}{\partial\sigma_3}+1\right)}$$

Final expression

$$\boxed{\sigma_n = \left\{\frac{\sigma_1 + \sigma_3\frac{\partial\sigma_1}{\partial\sigma_3}}{1+\frac{\partial\sigma_1}{\partial\sigma_3}}\right\}}$$

$$\sigma_n = \frac{\sigma_1 + \sigma_3\left[\frac{\partial\sigma_1}{\partial\sigma_3}+1-1\right]}{1+\frac{\partial\sigma_1}{\partial\sigma_3}}$$

Remember

$$\boxed{\sigma_n = \sigma_3 + \frac{\sigma_1-\sigma_3}{1+\frac{\partial\sigma_1}{\partial\sigma_3}}} \rightarrow (2)$$

put in eqn (1)

$$\tau_f^2 = \left(\frac{\sigma_1-\sigma_3}{2}\right)^2 - \left[\frac{\sigma_1+\sigma_3}{2} - \frac{\sigma_1+\sigma_3\frac{\partial\sigma_1}{\partial\sigma_3}}{1+\frac{\partial\sigma_1}{\partial\sigma_3}}\right]^2$$

UPCP

Date ___/___/___

$$\tau_f^2 = \left(\frac{\sigma_1-\sigma_3}{2}\right)^2 - \left[\frac{\sigma_1 + \sigma_1\frac{\partial\sigma_n}{\partial\sigma_3} + \sigma_3 + \sigma_3\frac{\partial\sigma_n}{\partial\sigma_3} - 2\sigma_1 - 2\sigma_3\frac{\partial\sigma_n}{\partial\sigma_3}}{2\left(1+\frac{\partial\sigma_n}{\partial\sigma_3}\right)}\right]^2$$

$$\tau_f^2 = \left(\frac{\sigma_1-\sigma_3}{2}\right)^2 - \left[\frac{\sigma_1\frac{\partial\sigma_1}{\partial\sigma_3} - \sigma_3\frac{\partial\sigma_n}{\partial\sigma_3} + \sigma_3 - \sigma_1}{2\left(1+\frac{\partial\sigma_n}{\partial\sigma_3}\right)}\right]$$

$$\tau_f^2 = \left(\frac{\sigma_1-\sigma_3}{2}\right)^2 - \frac{(\sigma_3-\sigma_3)}{2}\frac{\left[\frac{\partial\sigma_1}{\partial\sigma_3}-1\right]}{\left(1+\frac{\partial\sigma_1}{\partial\sigma_3}\right)}$$

$$\boxed{\tau_f = \frac{\sigma_{1f}-\sigma_{3f}}{1+\frac{\partial\sigma_{1f}}{\partial\sigma_{3f}}}\sqrt{\frac{\partial\sigma_{1f}}{\partial\sigma_{3f}}}}$$

$$\tan(\pi-2\theta_f) = \frac{\tau_f}{CA} = \frac{\tau_f}{\frac{\sigma_1+\sigma_3}{2}-\sigma_n}$$

$$\Rightarrow \tan(\pi-2\theta_f) = \frac{2(\sigma_1-\sigma_3)}{\left[1+\frac{\partial\sigma_{1f}}{\partial\sigma_{3f}}\right]\left[\sigma_1+\sigma_3-2\sigma_n\right]}\sqrt{\frac{\partial\sigma_{1f}}{\partial\sigma_{3f}}}$$

$$\tan(\pi-2\theta_f) = \left(\sqrt{\frac{\partial\sigma_{1f}}{\partial\sigma_{3f}}}\right)\frac{1}{\left(\frac{\partial\sigma_{1f}}{\partial\sigma_{3f}}-1\right)}$$

$$\tan 2\theta_f = \frac{2\sqrt{\partial\sigma_{1f}/\partial\sigma_{3f}}}{1-\frac{\partial\sigma_{1f}}{\partial_{3f}}}$$

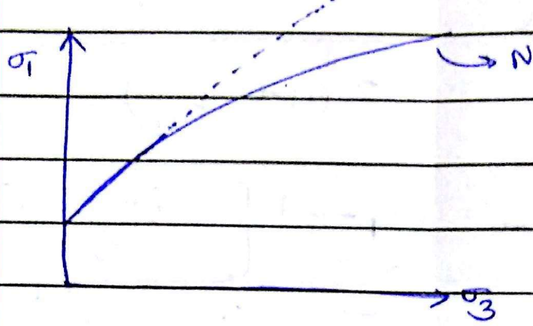
$$\tan(\theta_f+\theta_f) = \frac{\sqrt{\frac{\partial\sigma_{1f}}{\partial\sigma_{3f}}}+\sqrt{\frac{\partial\sigma_{1f}}{\partial\sigma_{3f}}}}{1-\sqrt{\frac{\partial\sigma_{1f}}{\partial\sigma_{3f}}}\sqrt{\frac{\partial\sigma_{1f}}{\partial\sigma_{3f}}}}$$

Date : ___/___/___

Remember all expression and try to derive it yourself.

$$\boxed{\tan\theta_f = \sqrt{\frac{\partial\sigma_1}{\partial\sigma_3}}}$$

2/1/17 **Non linear Strength Criteria :-**

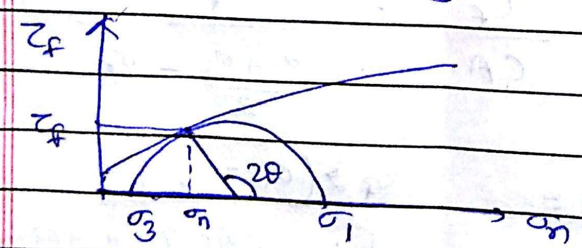


$$\sigma_{1f} = f(\sigma_{3f})$$
$$= f(\sigma_3)$$

at failure

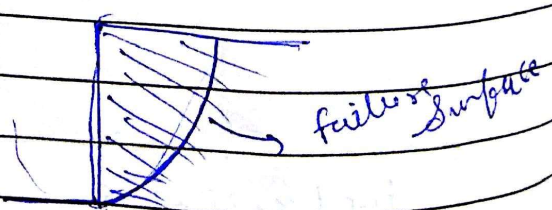
$$\sigma_n = \sigma_3 + \frac{\sigma_1 - \sigma_3}{1 + \frac{\partial\sigma_1}{\partial\sigma_3}}$$

$$\tau_f = \frac{\sigma_1 - \sigma_3}{1 + \frac{\partial\sigma_1}{\partial\sigma_3}} \sqrt{\frac{\partial\sigma_1}{\partial\sigma_3}}$$

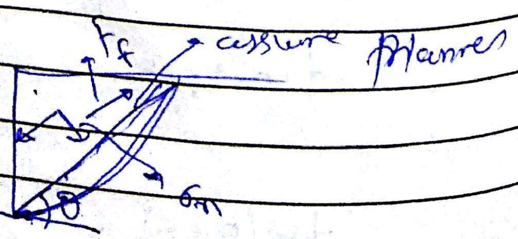


$$\tan\theta = \sqrt{\frac{\partial\sigma_1}{\partial\sigma_3}}$$

example Heavily fractured Rock Mass



assumption :- failure surface is circular



Date : ___/___/___

$F_f = \text{area} \times \tau_f$

$\sigma_{1f} = A\sigma_3^2 + B\sigma_3 + C$

$\frac{\partial \sigma_1}{\partial \sigma_3} = 2A\sigma_3 + B$

$$\tau_f = \left(\frac{\sigma_1 - \sigma_3}{1 + 2A\sigma_3 + B}\right)\sqrt{(2A\sigma_3 + B)}$$

$$\sigma_n = \sigma_3 + \frac{\sigma_1 - \sigma_3}{1 + 2A\sigma_3 + B}$$

first calculate σ_3 then τ_f

σ_n is known to me.

$\tau_f = f(\sigma_3)$ and $\sigma_3 = f(\sigma_n)$

If $\sigma_n = 5$ MPa what is $\tau_f = ?$

if $A = -0.02$, $B = 4$, $C = 6$

$$\sigma_n = 5 = \sigma_3 + \frac{(-0.02\sigma_3^2 + 4\sigma_3 + 6 - \sigma_3)}{1 - 0.04\sigma_3 + 4}$$

$25 - 0.2\sigma_3 = 5\sigma_3 - 0.04\sigma_3^2 - 0.02\sigma_3^2 + 3\sigma_3 + 6$

$\Rightarrow 0.06\sigma_3^2 - 8.2\sigma_3 + 19 = 0$

$\sigma_3 = 134.30$ and $\sigma_3 = 2.35$

$\sigma_1 = 182.47$ $\sigma_1 = 15.289$

so discard, $\cancel{\tau_f} = \frac{182.47 - 134.30}{1 - 0.04 \times 134.30 + 4}\sqrt{\underbrace{-0.04 \times 134.30 + 4}_{-ve}}$

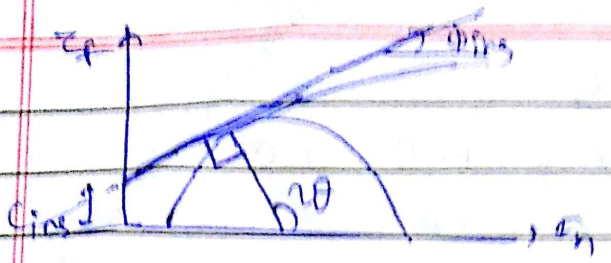
$$\tau_f = \frac{15.289 - 2.35}{1 - 0.04 \times 2.35 + 4}\sqrt{-0.04 \times 2.35 + 4}$$

$\boxed{\tau_f = 5.21 \text{ MPa}}$

$\tan\theta = \sqrt{-0.04 \times 2.35 + 4} \rightarrow 1.976$

UPCP

Date : ___/___/___



$$\tau_f = \sigma_n \tan(\phi_{inst}) + C_{inst}$$

$$2\theta = 90 + \phi_{inst}$$

$$\phi_{inst} = 2\theta - 90 \quad , \quad \boxed{\phi_{inst} = 36.32^\circ}$$

for, $\tan(\phi_{inst}) = \cancel{\tau_f}$

$$\boxed{C_{inst} = \tau_f - \sigma_n \tan(\phi_{inst})}$$

$$C_{ins} = 5.21 - 5\tan(36.32^\circ)$$

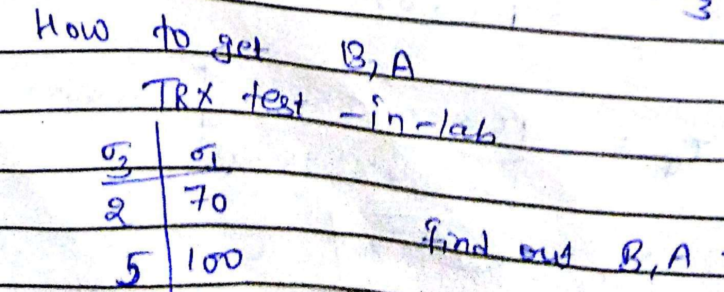
$$\boxed{C_{ins} = 1.534}$$

Empirical Criteria:-

a) Murrell (1965)

$$\sigma_1 = \sigma_c + B(\sigma_3)^A$$

(no need to Remember, But solve કરતાં આવડવા)



Parameters = B, A
not σ_c

How to get B, A

TRX test - in-lab

σ_3	σ_1
2	70
5	100
8	120
10	130

find out B, A = ?

Date : ___/___/___

Fit in Murell's model.

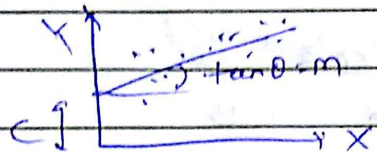
$$\sigma_1 - \sigma_c = B\,\sigma_3^{A}$$

$$\ln(\sigma_1 - \sigma_c) = \ln B + A \ln \sigma_3$$

$\sigma_c = UCS$

$$Y = \cancel{B} + AX$$

$$\boxed{Y = C + mX} \quad \text{linear}$$



$C = \ln B \Rightarrow B = e^C$

$m = A \Rightarrow A = m$

(b) Bleniawski

(no need to Remember)

$$\frac{\sigma_1}{\sigma_c} = 1 + B\left(\frac{\sigma_3}{\sigma_c}\right)^{\alpha}$$

$$\ln\left(\frac{\sigma_1 - \sigma_c}{\sigma_c}\right) = \ln B + \alpha \ln\left(\frac{\sigma_3}{\sigma_c}\right)$$

$$\boxed{Y = C + mX}$$

$B = e^C$, $\alpha = m$

$\alpha \simeq 0.74$ (more or less constant) $\Rightarrow$ B will be different for different plots

B depends on type of rock

	B
Mudstone / Silt Stone	3
Sand Stone	4
Quartzite	4.5
Norite	5

(c) Hoek-Brown (1980)

Remember exam

most widely used

UPCP

$$\sigma_1 = \sigma_3 + \sqrt{m\,\sigma_{ci}\,\sigma_3 + s\,\sigma_{ci}^2}$$

σ_{ci} = intact rock UCS ← It is Parameter Here.

Date : ___/___/___

Parameters $= m, \sigma_{ci}, S$

For intact Rocks

$S = 1$

for intact Rocks $\leftarrow$ $$\boxed{\sigma_1 = \sigma_3 + \sqrt{m\sigma_{ci}\sigma_3 + \sigma_{ci}^2}}$$

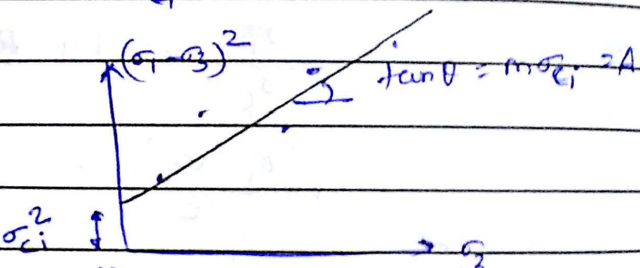
$$(\sigma_1 - \sigma_3)^2 = m\sigma_{ci}\sigma_3 + \sigma_{ci}^2$$

$$Y = AX + B$$

$$A = m\sigma_{ci}$$

$$B = \sigma_{ci}^2$$

$Y = (\sigma_1 - \sigma_3)^2$, $X = \sigma_3$



σ_3	σ_1
2	70
5	100
8	120
10	130
0	40
0	50

→ Total 6 data Point ≥ 1

→ for Murell $\Rightarrow \sigma_c = \frac{40+50}{2} \Rightarrow 45$ (it is not Parameter) (only 4 data Point required) for Best fit

→ But for Hoek brown, it is Parameter

→ Take all data Point (6 data Points)

σ_{ci} is Parameter now. (dont take avg)

σ_3	σ_1	$Y = (\sigma_1 - \sigma_3)^2$	$X = \sigma_3$
0	40	1600	0
0	50	2500	0
2	70	4624	2
5	100	9025	5
8	120	12544	8
10	130	14400	10

$B = 2171.90$

$A = 1266.46$

UPCP

~~$Y = 1266.46x + 2171.90$~~

$\boxed{\sigma_{ci} = 46.60}$

$\boxed{m = 27.17}$

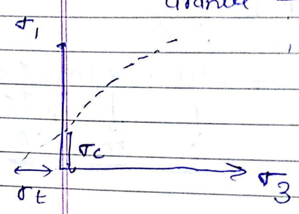
7/3/17 (✓)

7/03/17

Rough value of m.

Greas – 24.5

Granite – 29.2



Physical significance of m.

For $\sigma_3 = -\sigma_t$

$\sigma_1 = 0$

$$0 = -\sigma_t + \sqrt{m \cdot \sigma_c(-\sigma_t) + \sigma_c^2}$$

$$\sigma_t^2 = \sigma_c^2 - m\sigma_c\sigma_t$$

Remember

$$\boxed{m = \frac{\sigma_c}{\sigma_t} - \frac{\sigma_t}{\sigma_c}}$$

$$\boxed{m \simeq \frac{\sigma_c}{\sigma_t}}$$

IITD ⇢

Ramamurthy Criterion ⇢

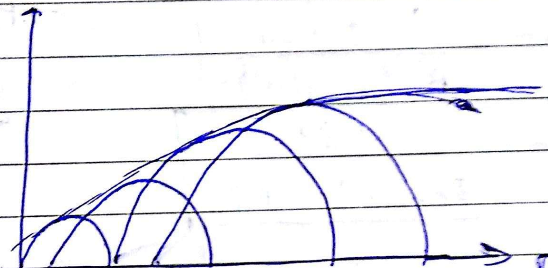
$$\left(\frac{\sigma_1-\sigma_3}{\sigma_3}\right) = \beta i \left(\frac{\sigma ci}{\sigma_3}\right)^{\alpha i}$$

Parameter → $\beta i, \alpha i$ [average σc]

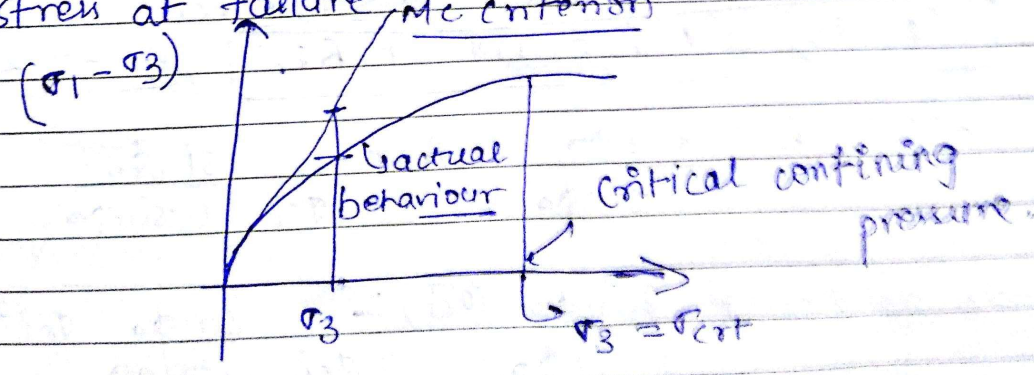
$$\underbrace{\ln\left(\frac{\sigma_1-\sigma_3}{\sigma_3}\right)}_{y} = \underbrace{\log(\beta i)}_{c} + \underbrace{\alpha i}_{m} \underbrace{\ln\left(\frac{\sigma ci}{\sigma_3}\right)}_{x}$$

IITR ⇢

Modified mohr-coloumb ~~failure~~ criterion ⇢



- If confining pressure is increased, the differential stress at failure increases with increase in confining pressure, rate of increase of differential stress reduces and at a sufficiently high confining pressure there is no further increase in differential stress at failure



$$\sigma_1 = \frac{2\, c_0 \cos\phi_0}{1-\sin\phi_0} + \frac{1+\sin\phi_0}{1-\sin\phi_0}\sigma_3$$

At $\sigma_3 \approx 0$, $C = c_0$, $\phi = \phi_0$.

$$\sigma_1 - \sigma_3 = \frac{2 c_0 \cos\phi_0}{1-\sin\phi_0} + \left(\frac{1+\sin\phi_0}{1-\sin\phi_0}\right)\sigma_3 - \sigma_3$$

Date: / /201

$$\sigma_1-\sigma_3 = \frac{2c_0\cos\phi_0}{1-\sin\phi_0} + \frac{2\sin\phi_0}{1-\sin\phi_0}\sigma_3$$

Non-linear criterion (MMC)

$$(\sigma_1-\sigma_3) = \frac{2c_0\cos\phi_0}{1-\sin\phi_0} + \frac{2\sin\phi_0}{1-\sin\phi_0}\sigma_3 - A'\sigma_3^2$$

$$\left.\frac{\partial(\sigma_1-\sigma_3)}{\partial\sigma_3}\right|_{\sigma_3=\sigma_{crt}} = \frac{2\sin\phi_0}{1-\sin\phi_0} - 2A'\sigma_3 = 0$$

$$\frac{\sin\phi_0}{1-\sin\phi_0} = A'\sigma_3$$

$$\sigma_{crt} = \frac{1}{A'}\left[\frac{\sin\phi_0}{1-\sin\phi_0}\right]$$

Experimentally

$$\boxed{\sigma_{crt} \simeq \sigma_{ci}}$$

$$\boxed{A' = \frac{\sin\phi_0}{\sigma_{crt}(1-\sin\phi_0)}}$$

$$\boxed{A' = \frac{\sin\phi_0}{\sigma_{ci}(1-\sin\phi)}}$$

Eq

$$\boxed{\sigma_1-\sigma_3 = \frac{2c_0\cos\phi_0}{1-\sin\phi_0} + \frac{2\sin\phi_0}{1-\sin\phi_0}\sigma_3 - \frac{1}{\sigma_{ci}}\frac{\sin\phi_0}{(1-\sin\phi_0)}\sigma_3^2}$$

OR $0 \le \sigma_3 \le \sigma_{ci}$

$$\boxed{(\sigma_1-\sigma_3) = \sigma_{ci} + B\sigma_3 + A\sigma_3^2}, \quad \boxed{B = -2\sigma_{ci}A}$$

where $B = \frac{2\sin\phi_0}{1-\sin\phi_0}$, $A = \frac{1}{\sigma_{ci}}\frac{\sin\phi_0}{1-\sin\phi_0}$

for $\sigma_3 > \sigma_{ci}$

$$\boxed{\sigma_1-\sigma_3 = \sigma_{ci} + \frac{2\sin\phi_0}{1-\sin\phi_0}(\sigma_{ci}) - \frac{1}{\sigma_{ci}}\frac{\sin\phi_0}{1-\sin\phi_0}\sigma_{ci}^2}$$

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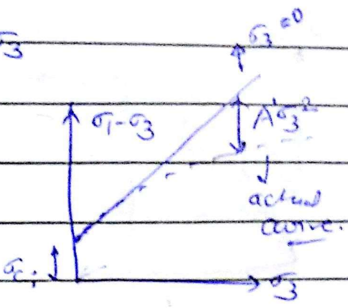
Strength Criteria:-

MMC (Modified Mohr Coulomb Criteria)

$$\sigma_1 = \frac{2C_0\cos\phi_0}{1-\sin\phi_0} + \frac{1+\sin\phi_0}{1-\sin\phi_0}\sigma_3$$

Subtracting σ_3 (LHS and RHS)



$$\sigma_1 - \sigma_3 = \frac{2C_0\cos\phi_0}{1-\sin\phi_0} + \frac{2\sin\phi_0}{1-\sin\phi_0}\sigma_3$$

Proposed Equation

$$\sigma_1 - \sigma_3 = \sigma_{c_i} + B\sigma_3 - A'\sigma_3^2$$

$$\sigma_{c_i} = \frac{2C_0\cos\phi_0}{1-\sin\phi_0} \qquad B = \frac{2\sin\phi_0}{1-\sin\phi_0}$$

0 → indicates $\sigma_3 \to 0$

another way of Representation

exam
$$\boxed{(\sigma_1 - \sigma_3) = \sigma_{c_i} + B\sigma_3 + A\sigma_3^2}$$

Boundary Condition

from experience $\sigma_{3\,cut} = \sigma_{c_i}$

$$\frac{\partial(\sigma_1 - \sigma_3)}{\partial\sigma_3} = 0 + B + 2A\sigma_3$$

at $\sigma_3 \to \sigma_{3_{c_i}} = \sigma_{c_i}$, $\frac{\partial(\sigma_1 - \sigma_3)}{\partial\sigma_3} \to 0$

So,
$$B + 2A\sigma_{3_{c_i}} = 0$$

$$B + 2A\sigma_{c_i} = 0$$

$$\boxed{B = -2A\sigma_{c_i}}$$

So, for the range $\sigma_3 \le \sigma_{c_i}$

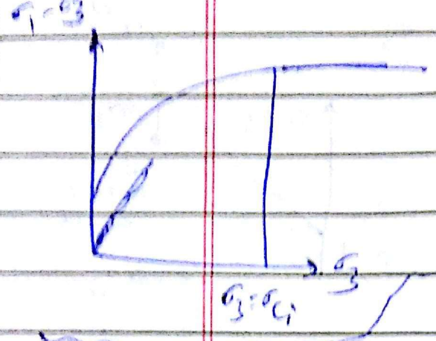
$$\boxed{(\sigma_1 - \sigma_3) - \sigma_{c_i} - 2A\sigma_{c_i}\sigma_3 + A\sigma_3^2}$$

A = empirical Coefficient

For $\sigma_3 > \sigma_{c_i}$

For $\sigma_3 = \sigma_{c_i}$ $\quad (\sigma_1 - \sigma_3) = \sigma_{c_i} - 2A\sigma_{c_i}\sigma_{c_i} + A\sigma_{c_i}^2$

$$\boxed{(\sigma_1 - \sigma_3) = \sigma_{c_i} - A\sigma_{c_i}^2}$$



$$A = \frac{-1}{\sigma_{c_i}} \frac{\sin\phi_0}{(1 - \sin\phi_0)}$$

$$\boxed{\sigma_3 < \sigma_{c_i}} \Rightarrow \underbrace{(\sigma_1 - \sigma_3 - \sigma_{c_i})}_{Y} = A\underbrace{(\sigma_{c_i}^2 - 2\sigma_{c_i}\sigma_3)}_{X}$$

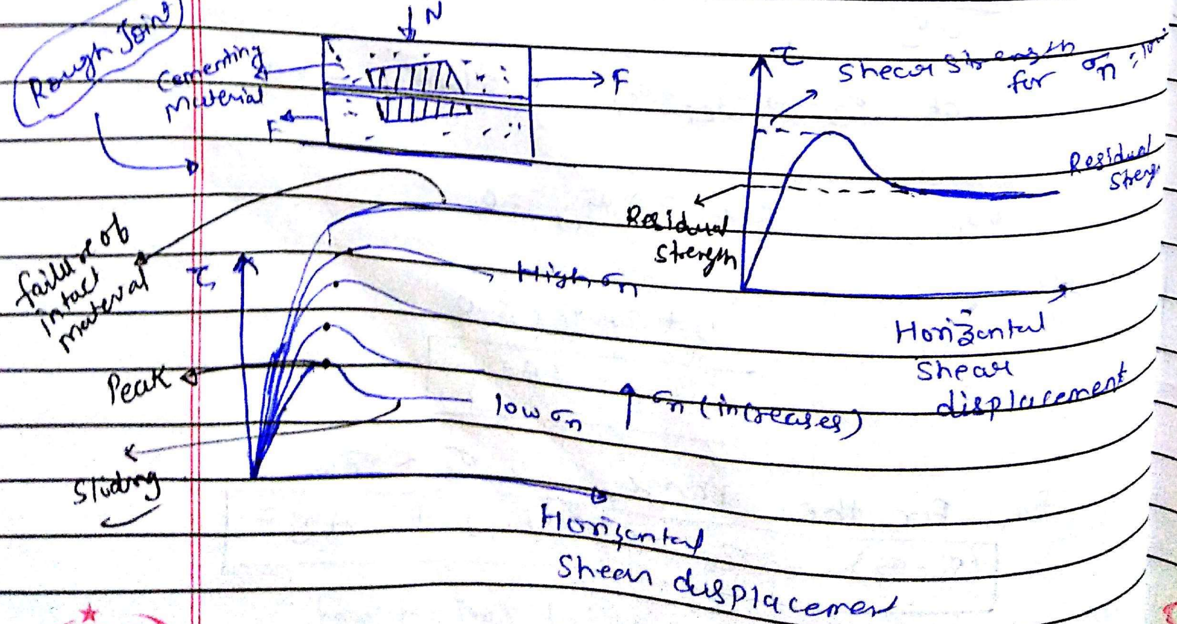
$$\boxed{Y = AX}$$

$$\boxed{A = \frac{\sum(\sigma_1 - \sigma_3 - \sigma_{c_i})}{\sum(\sigma_{c_i}^2 - 2\sigma_{c_i}\sigma_3)}}$$

Shear Strength Behaviour of Rock Joints :-

~~lets us say~~

lets we have Jointed Rock



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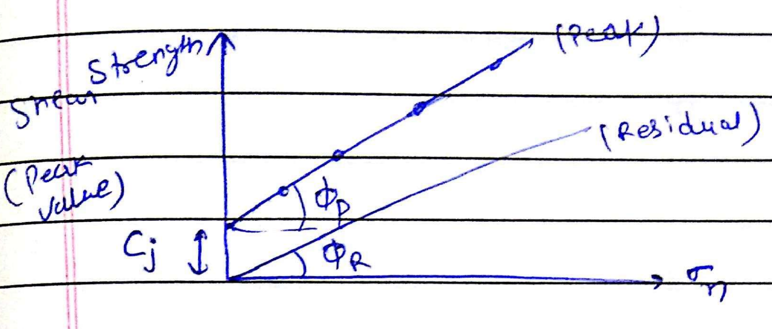
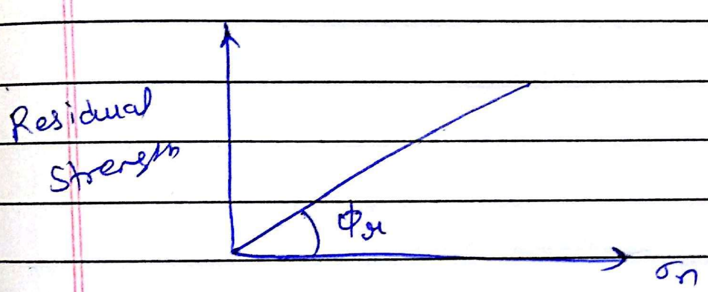
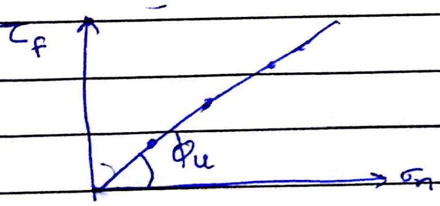


assumption:- linear behaviour



Shear Strength Models :-

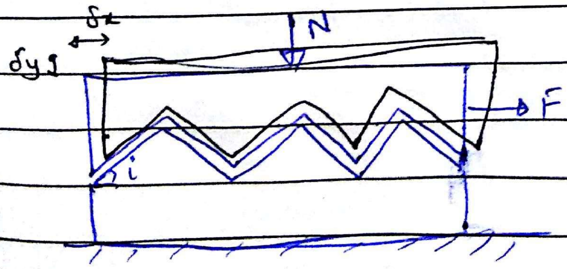
Saw Cut Joint (Smooth joint)



ϕ_u = "friction coefficient" $\boxed{\mu = \tan\phi_u}$

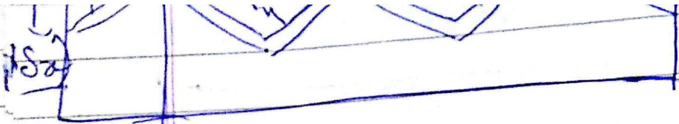
intercept = 0 (for smooth surface)

intercept $\neq 0$ (Rough surface)



N = Low ⇒ Pure sliding (But thoda sahut shearing ~~sho~~ (hi))

$\delta x, \delta y$ displacement



N = Low - SL

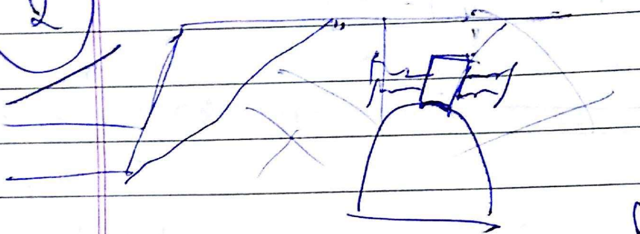
10/03/17 **Shear strength of Rock Joints :->**

CNL – Constt. Normal Load Condition. → Normal load constant during shear

CNS – Constt. Normal stiffness condition.

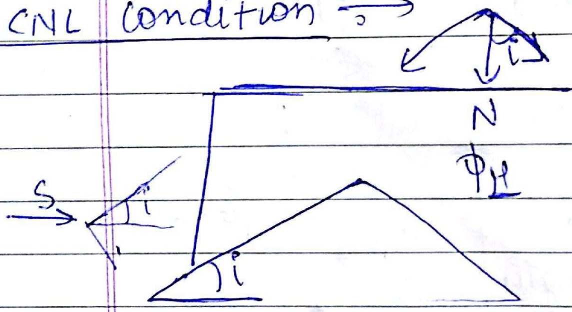
↳ will change with displacement (enhanced strength)

(2)



Patton's Model :->

CNL condition :->



N = Low = Only sliding

Component causing sliding
$= S\cos i - N\sin i$

Normal to sliding surface
$= S\cdot\sin i + N\cos i$

At the verge of sliding

$\tau = \sigma_n \tan\phi_\mu$ — Basic ~~pr~~ friction angle.

$$\frac{S\cos i - N\sin i}{AB} = \frac{S\sin i + N\cos i}{AB} \times \tan\phi_\mu$$

$$S(\cos i - \tan\phi_\mu \sin i) = N(\sin i + \cos i \times \tan\phi_\mu)$$

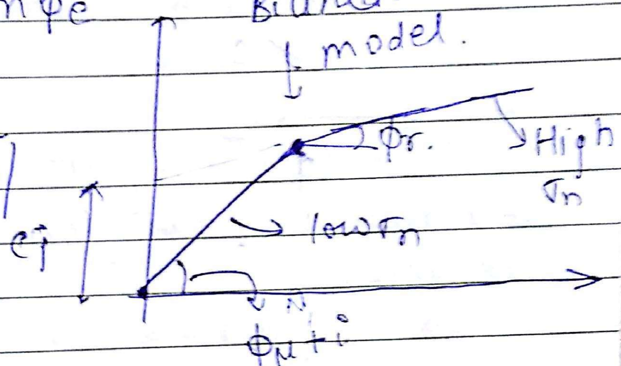
$$\boxed{S = N\cdot\tan(\phi_\mu + i)}$$

$$\boxed{\tau_f = \sigma_n \cdot \tan(\phi_\mu + i)}$$ → For low normal load.

$\tau_f = \sigma_n \tan\phi_e$

For high σ_n :→

$$\boxed{\tau_f = c_j + \tan(\phi_r)\cdot\sigma_n}$$

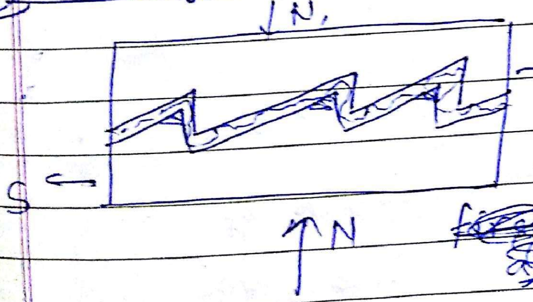


1) Coulomb's Equation :→

$$\boxed{\tau_f = c_j + \sigma_n \tan\phi_j}$$

Angle of reproach is limiting value of ϕ when σ_n tends to zero.

3) Ladangi & Archambault :→



During shear :→

Shear force, S will comprise of the following components.

① S_1 = This is the component due to external work in dilating against the external force N

② S_2 = Additional internal work done in friction, due to dilation only.

③ S_3 = Component due to work done in internal friction if the specimen does not change in volume.

S_4 = work done in shearing the asperities.

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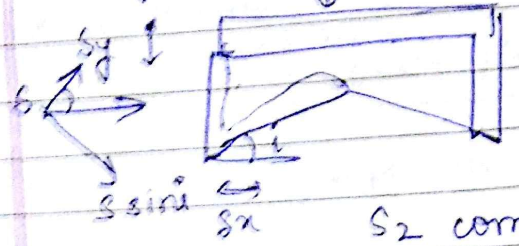
Ladanyi, Archambault – based on energy principles, shear strength model for rock joint

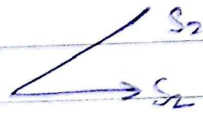
$S_1 \delta x = N \delta y$



$S_1 = \frac{N \delta y}{\delta x} = N \tan i$ — ①

S_2 component

$(S \sin i) \times \tan\phi_u$ = Frictional force $= S_2 \cos i$



$S_2 = S \tan i \tan\phi_u$ — ②.

First 2 components ∵ of geometry

If dilation was not there, first two components will be zero.

$S_3 = N \tan\phi_u$

No shearing → Only sliding → $S_4 = 0$.

$S = S_1 + S_2 + S_3 + S_4$

$S = N \tan i + S \tan i \tan\phi_u + N \tan\phi_u$.

$$S = \frac{N(\tan i + \tan\phi_u)}{1 - \tan i \tan\phi_u}$$

$$\boxed{S = N \tan(\phi_u + i)}$$

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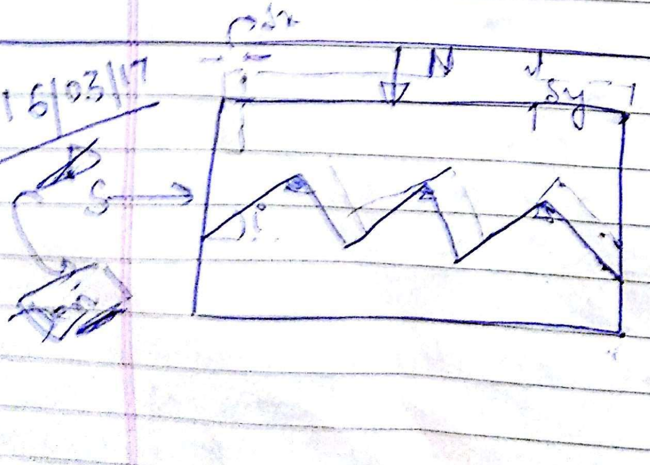

$S = S_1 + S_2 + S_3 + S_4$

$S_1 dx = N_1 dy$

$S_1 = N \tan i$

$S_2 = S \tan i \tan\phi_u$

$S_3 = N \tan\phi_u$

S_4 = Shear = Mohr Coulomb Equation

$\tau = c_i + \sigma_n \tan\phi_i$ for intact material ~~plane~~ failure (c_i, ϕ_i)

$S_4 = A(c_i + \sigma_n \tan\phi_i)$

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$$S = (S_1 + S_2 + S_3)(1 - a_s) + S_4 \cdot a_s$$

a_s = Shear Area Ratio $= \frac{A_s}{A}$

$$S = (N \tan i + S \cdot \tan i \cdot \tan\phi_u + N \tan\phi_u)(1 - a_s) + (A)(c_i + \sigma_n \tan\phi_i)\, a_s.$$

$$S - S \tan i \cdot \tan\phi_u (1 - a_s) = (N \tan i + N \tan\phi_u)(1 - a_s) + A(c_i + \sigma_n \tan\phi_i)(a_s).$$

$$S = \frac{[N(\tan i + \tan\phi_u)(1 - a_s) + A(c_i + \sigma_n \tan\phi_i)(a_s)]}{[1 - \tan i \tan\phi_u (1 - a_s)]}$$

$$\boxed{\tau = \frac{S}{A} = \frac{[\sigma_n (1 - a_s)(\tan i + \tan\phi_u) + (c_i + \sigma_n \tan\phi_i)\, a_s]}{[1 - \tan i \tan\phi_u (1 - a_s)]}}$$

R

If σ_n is very very low.
$\rightarrow a_s \simeq 0$

$$\boxed{\tau = \frac{\sigma_n (\tan i + \tan\phi_u)}{(1 - \tan i \tan\phi_u)}} \Rightarrow \boxed{\tau = \sigma_n \tan(\phi_u + i)}$$

If σ_n is very very large

$$\boxed{\tau = c_i + \sigma_n \tan\phi_i}$$

$$S \cdot \delta x = \left[N \delta y + S \cdot \delta_m \times \tan\phi_u \times \frac{\delta x}{\cos i} + N \cdot \tan\phi_u \, \delta x\right](1 - a_s) + [A(c_i + \sigma_n \tan\phi_i)]\, a_s \cdot \delta x$$

NR

$$a_s = 1 - \left(1 - \frac{\sigma_n}{\sigma_{TRN}}\right)^{k_1}$$

$k_1 \rightarrow 1.5$

$\sigma_{TRN} \rightarrow \simeq \sigma_{cs}$

$$\tan i = \left(1 - \frac{\sigma_n}{\sigma_{TRN}}\right)^{k_2} \cdot \tan i_0$$

$\sigma_{TRN} \rightarrow$ Transition stress (Brittle to Ductile)

a_s is a function of Normal stress

Barton's Model $\rightarrow$ (Most widely used)

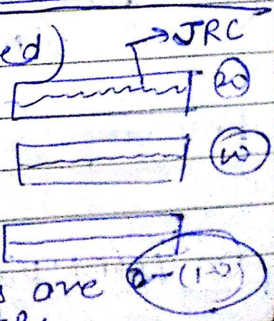
- Tests on specimen with different roughness and normal stresses.

JRC $\rightarrow$ Joint Roughness Coefficient.

$$\boxed{\tau = \sigma_n \tan\left[\phi_r + JRC \log_{10} \frac{JCS}{\sigma_n}\right]}$$

ϕ_r (degree)

Standard profiles are availabel.



JCS → Joint wall compressive strength.
If fresh joint → equal to UCS
~~(Schmidt hammer test)~~

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Unit

Previous claim → $\tau = \sigma_n(\tan\phi_r + i)$
b, ϕ_μ

$\boxed{JRC \rightarrow \text{in degree } (i_0)}$

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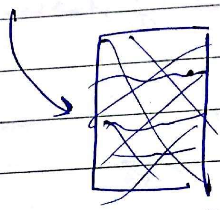
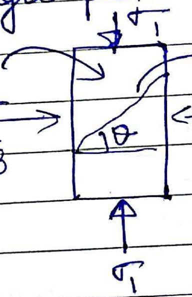
Intact Rock Strength :→

Rock Joint Strength (Interface)
(Discontinuity strength)

176 (Jointed Rock | Rock Mass strength)



Single plane of weakness Theory → (Jaeger 1960)



$\boxed{c_j \le c_i}$

$\sigma_{1f} = f(\sigma_3, P)$
Known → c_i, ϕ_i

Shear strength b/w of joint $= \tau_f$
$$= \sigma_n \tan\left\{\phi_u + JRC \log_{10}\frac{JCS}{\sigma_n}\right\}$$

$$\tau_f = c_j + \sigma_n \tan\phi_j$$

(I) Stress required to cause failure through intact rock $= \dfrac{2c_i\cos\phi_i}{1-\sin\phi_i} + \dfrac{1+\sin\phi_i}{1-\sin\phi_i}\cdot\sigma_3$

(II) Stress (σ_1) required to cause failure along joint plane
$$\tau_f = c_j + \sigma_n\tan\phi_j$$
at the time of failure, $\sigma_1 = \sigma_{1f}$.

Resolving :→ $\sigma_n = \left(\dfrac{\sigma_1+\sigma_3}{2}\right) + \left(\dfrac{\sigma_1-\sigma_3}{2}\right)\cos 2\theta.$

$\tau = \dfrac{\sigma_1-\sigma_3}{2}\sin 2\theta$, , failure occurs when $\tau \ge \tau_f$

On joint plane :→ $\left(\dfrac{\sigma_1-\sigma_3}{2}\right)\cdot\sin 2\theta = c_j + \left\{\dfrac{\sigma_1+\sigma_3}{2} + \dfrac{\sigma_1-\sigma_3}{2}\cos 2\theta\right\}\tan\phi_j$

(R)

$$\sigma_{1f} = \sigma_3 + \frac{2(c_j + \sigma_3 \tan\phi_j)}{\sin 2\theta\left(1 - \frac{\tan\phi_j}{\tan\theta}\right)}$$

→ strength of jointed rock would be smaller if (I) & (II)

$$\frac{\partial(\sigma_{1f})}{\partial\theta} = 0 \Rightarrow 2\cos 2\theta\left(1 - \frac{\tan\phi_j}{\tan\theta}\right) + \sin 2\theta\left(0 + \operatorname{cosec}^2\theta \tan\phi_j\right) = 0.$$

$$\Rightarrow \underset{\times\tan\theta}{2\cos 2\theta} - \left(\frac{2\cos 2\theta}{\tan\theta}\right)\tan\phi_j + \frac{2\sin\theta\cos\theta}{\sin^2\theta}\tan\phi_j = 0$$

$$2(\cos 2\theta)(\tan\theta) = \tan\phi_j(-2 + 2\cos 2\theta)$$

$$\tan\phi_j(\cos 2\theta - 1) = (\cos 2\theta)\tan\theta.$$

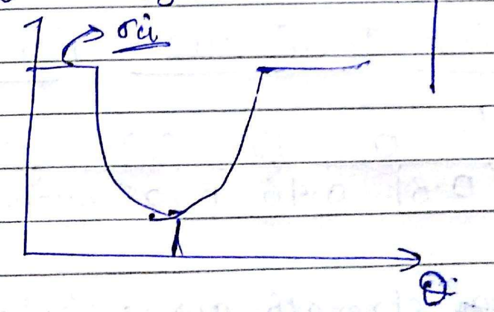
$$\tan\phi_j \cdot \tan 2\theta = -1$$

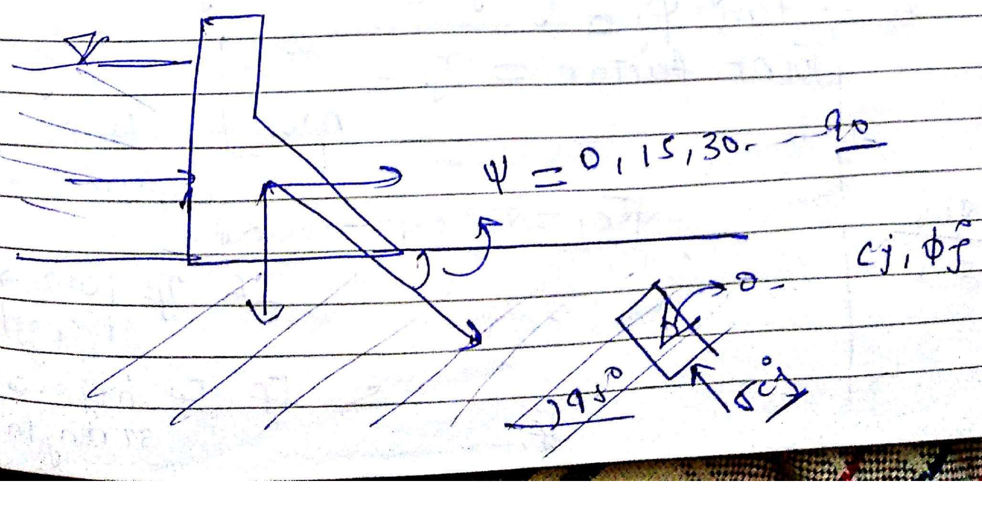
$$2\theta = 90 + \phi_j \rightarrow \boxed{\theta = 45 + \frac{\phi_j}{2}}$$

$$\sigma_{cj} = \frac{2c_j}{\sin 2\theta\left(1 - \frac{\tan\phi_j}{\tan\theta}\right)}$$

ex $c_j = 0.5\,MPa$, $\phi_j = 30°$, $\sigma_3 = 0$

θ	σ_j



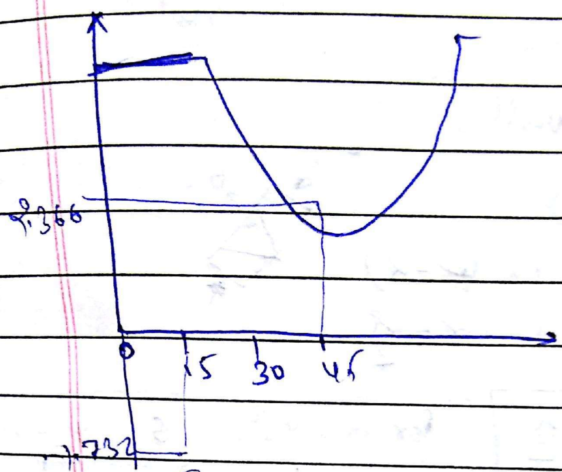


Example $c_j = 0.5$ MPa
$\phi_j = 30^\circ$

~~$\sigma_{c_j} \Rightarrow$ ( ) ($\sigma_3 = 0$) UCS~~

$\Rightarrow$

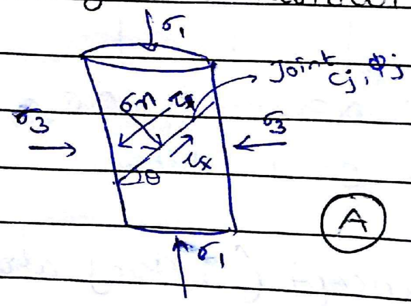
$$\sigma_{c_j} = \frac{2c_j}{\sin 2\theta\left(1 - \frac{\tan\phi_j}{\tan\theta}\right)}$$



B)

θ	σ_{c_j}	
15	−1.732	(Tension)
45	2.366	(Compressive)
60	1.732	
75	2.366	

21/3/17 **Strength Behaviour of Jointed Rocks**



For failure $\tau \geq \tau_f$

(A) Single plane of weakness Theory :-

① σ_1 – failure along the joint plane

② σ_1 – failure of intact material

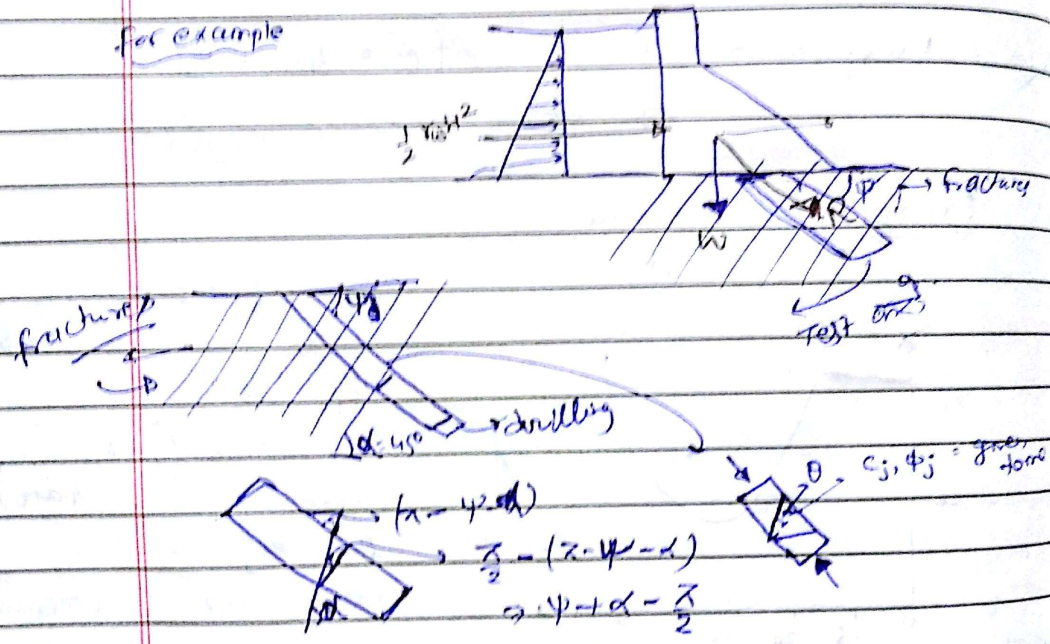
Strength of Rock $\Leftarrow$ Physically minimum

$$\sigma_1 = \sigma_3 + \frac{2(c_j + \sigma_3 \tan\phi_j)}{\sin 2\theta \left(1 - \frac{\tan\phi_j}{\tan\theta}\right)}$$

when $\sigma_3 = 0$ $\quad (\sigma_1 = UCS = \sigma_{cj})$

$$\boxed{\sigma_{cj} = \frac{2c_j}{\sin 2\theta \left(1 - \frac{\tan\phi_j}{\tan\theta}\right)}}$$

for example



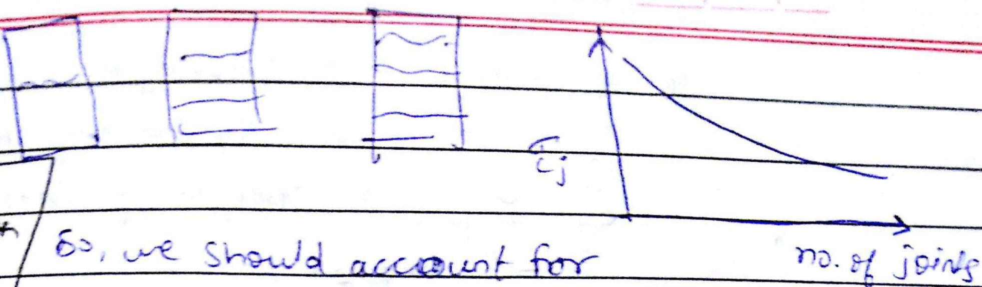
$\boxed{\theta = \psi + \alpha - \frac{\pi}{2}}$ for $\alpha = 45^\circ$, $\boxed{\theta = \psi - 45^\circ}$

ψ	θ_1	θ_2	σ_{c1}	σ_{c2}	σ_{cj}	Select minimum

assumption: No Interference of planes with each other.

(B) Joint factor concept (Talking about UCS only)

1) frequency of joint is important

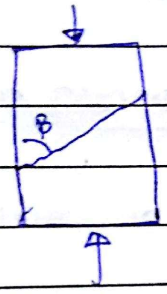


no. of joints per meter depth $= J_n$

So, we should account for frequency of joints.

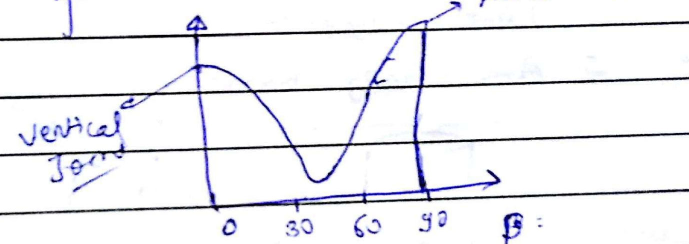
2) orientation is important

β	n
0	0.81
10	0.46
20	0.105
30	0.046
40	0.071
50	0.306
60	0.465
70	0.634
80	0.814
90	1.0



at what β, you will get minimum UCS

$\beta = 45 - \phi/2$



n → joint inclination parameter

$$\boxed{\text{for } \beta,\ \text{strength} = n \text{ times of UCS}}$$

3) shear strength along joint $= \phi_j$ (roughness)

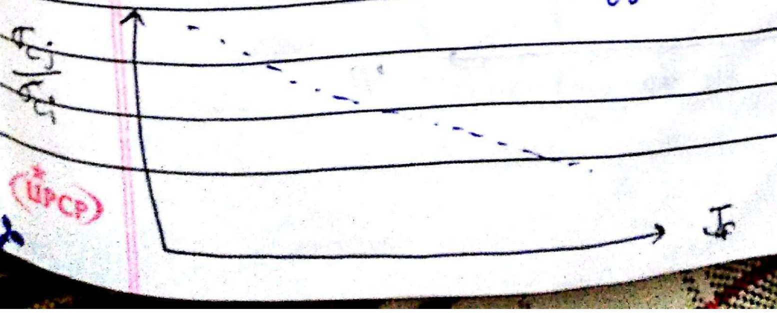
$r = \tan\phi_j$

$$\boxed{\text{joint factor } (J_f) = \frac{J_n}{n \cdot r}}$$

J_n → No. of joints per meter depth

r → $\tan\phi_j$

n → for different β, there is a 'n'



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if $J_f \Rightarrow$ High $\Rightarrow$ Strength = low

J_f = low $\Rightarrow$ Strength = High

$J_f = 0$ $\Rightarrow$ Strength of intact Rock

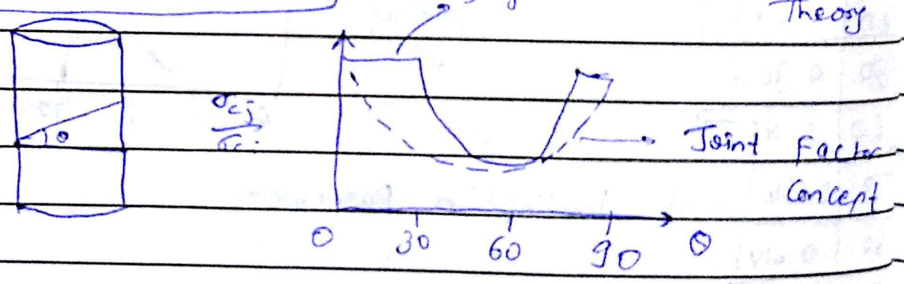
(no need to remember) $$\boxed{\sigma_{cj} = \sigma_{ci}\, e^{-(0.008 J_f)}}$$

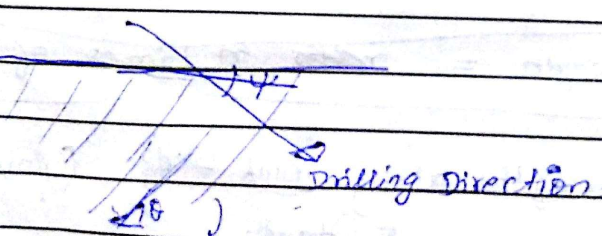
Upper limit of joints Strength = UCS of intact Rock

due to defects strength has decreased.

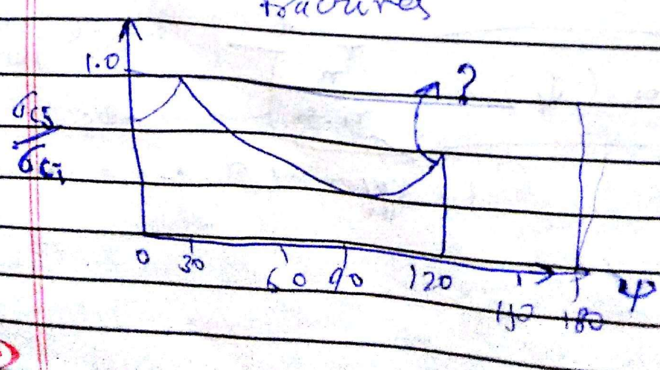
23/3/17 Strength Behaviour of Jointed Rocks and Rock Masses :-

$\dfrac{\sigma_{c_{max}}}{\sigma_{c_{min}}}$ = Anisotropy Ratio :-





$J_f = \frac{J_n}{n \cdot r}$



Ramamurthy :- (for jointed Rocks)

$$\left(\frac{\sigma_{1f}-\sigma_3}{\sigma_3}\right) = B_j\left(\frac{\sigma_{cj}}{\sigma_3}\right)^{\alpha_j}$$

no need to remember

$$\boxed{\alpha_j = \left(\frac{\sigma_{cj}}{\sigma_{ci}}\right)^{0.5} \times \alpha_i} \rightarrow \text{TRx test intact}$$

$$\boxed{B_j = \frac{B_i}{0.13\, e^{\{2.03\,\alpha_j/\alpha_i\}}}}$$

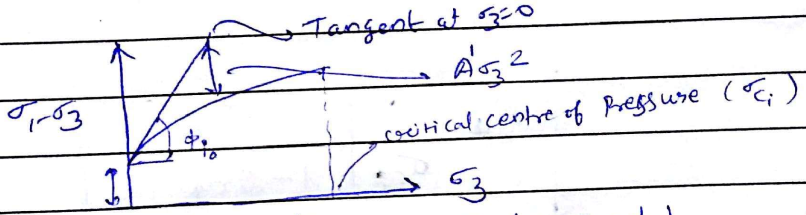
IITR

Singh + Singh (2005) RMRE

Singh et al — 2011

— 2012

MMC :- Intact Rock :-



$$\sigma_1 = \frac{(2\cos\phi_i)(c)}{1-\sin\phi} + \sigma_3\frac{(1+\sin\phi)}{(1-\sin\phi)}$$

ϕ A

ग्रहण किया था

ϕ_{i0}

Tangent at $\sigma_3=0$

$$(\sigma_1-\sigma_3) = \frac{2c\cos\phi}{1-\sin\phi} + \frac{2\sin\phi}{1-\sin\phi}\sigma_3$$

$$\boxed{(\sigma_1-\sigma_3) = A\sigma_3^2 + B\sigma_3 + \sigma_{ci}}$$

$$\underbrace{(\sigma_1-\sigma_3-\sigma_{ci})}_{Y} = A\,\underbrace{(\sigma_3^2-2\sigma_{ci}\sigma_3)}_{X}$$

$$A = \frac{\Sigma(\sigma_1-\sigma_3-\sigma_{ci})}{\Sigma(\sigma_3^2-2\sigma_{ci}\sigma_3)}$$

UPCP

Date : / /

$B = -2A\sigma_{c_i}$

$B = \dfrac{2\sin\phi_{j_0}}{1-\sin\phi_{j_0}} \Rightarrow \boxed{\sin\phi_{j_0} = \dfrac{B}{2+B}}$

ϕ_{j_0} is a parameter

Example

For Jointed Rocks

$$(\sigma_1-\sigma_3) = \sigma_{c_j} + \frac{2\sin\phi_{j_0}}{1-\sin\phi_{j_0}}\sigma_3 - A_j'\sigma_3^2$$

no need to remember but understand

$$\frac{\partial(\sigma_1-\sigma_3)}{\partial\sigma_3} = 0 \Rightarrow \frac{2\sin\phi_{j_0}}{1-\sin\phi_{j_0}} = 2A_j'\sigma_3 \quad \text{for } \sigma_3 = \sigma_{c_i}$$

28/3/1

$$\Rightarrow \frac{2\sin\phi_{j_0}}{1-\sin\phi_{j_0}} = 2A_j'\sigma_{c_i}$$

$$\boxed{A_j' = \frac{1}{\sigma_{c_i}}\left[\frac{\sin\phi_{j_0}}{1-\sin\phi_{j_0}}\right]}$$

$$\boxed{(\sigma_1-\sigma_3) = \sigma_{c_j} + \frac{2\sin\phi_{j_0}}{1-\sin\phi_{j_0}}\sigma_3 - \frac{1}{\sigma_{c_i}}\left[\frac{\sin\phi_{j_0}}{1-\sin\phi_{j_0}}\right]\sigma_3^2}$$

ex: Basalt ⇐

Granite

To apply theory σ_{c_j} and ϕ_{j_0} is unknown

To get ϕ_{j_0}

For jointed rocks: $(\sigma_1-\sigma_3)_{max}$, Put $\sigma_3 = \sigma_{c_i}$

$$(\sigma_1-\sigma_3)_{max} = \sigma_{c_j} + \frac{2\sin\phi_{j_0}}{1-\sin\phi_{j_0}}\sigma_{c_i} - \left[\frac{\sin\phi_{j_0}}{1-\sin\phi_{j_0}}\right]\sigma_{c_i}$$

$$= \sigma_{c_j} + \frac{\sin\phi_{j_0}}{1-\sin\phi_{j_0}}\sigma_{c_i} \quad \textcircled{1}$$

UPCP

for intact Rocks

$$(\sigma_1 - \sigma_3)_{max} = \sigma_{ci} + \frac{\sin\phi_{i_0}}{1-\sin\phi_{i_0}} \sigma_{ci} \rightarrow ②$$

eqⁿ ① and ② are equal

$$\boxed{\sin\phi_{j_0} \Rightarrow \frac{\left\{\left(1-\frac{\sigma_{cj}}{\sigma_{ci}}\right) + \frac{\sin\phi_{i_0}}{1-\sin\phi_{i_0}}\right\}}{\left\{\left(2-\frac{\sigma_{cj}}{\sigma_{ci}}\right) + \frac{\sin\phi_{i_0}}{1-\sin\phi_{i_0}}\right\}}}$$

• no need to remember but understand

28/3/17 Classification of Rocks and Rock Masses

Intact Rock :-

Geological classification:-

Tells about "origin"

Igneous → formed due to cooling and Solidification of molten mass (Magma)

extrusive → Solidifies ⇒ Cooling (fast cooling) ⇒ Crystallization Incomplete ⇒ ex: Basalt

Intrusive → Cooles ⇒ Beneath earth surface (gradual) ⇒ Sufficient crystallization ⇒ Granite

Sedimentary: sediments consolidated

① weathering or erosion
② Transportation
③ Deposition
④ Consolidation and Cementation

ex: Sandstone, Siltstone

or

$\sigma_3 = \sigma_{ci}$

$\frac{\phi_{j_0}}{\sin\phi_{j_0}} \sigma_3^2$

unknown

$\frac{\sin\phi_{j_0}}{1-\sin\phi_{j_0}} \sigma_{ci}$ → ①

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Metamorphic :-

Heat, Pressure, Chemical → Rock → Change → Texture, Structure, Mineral Cont → Recrystallization

for ex :- Quartzite

Engineering Classification :- (Rock Engineering)

Purpose

1) Identify "Most significant Parameters"
2) Divide into classes
3) Experience at one site ⇒ Site Conditions at other site.
4) Derive "quantitative" data and guidelines for eyes design – numbers / strength / E = ?

~~Factors Affecting~~

Significant Parameters :-

1. Colour
2. Minerology
3. Weathering / Alteration
4. Density
5. Void Ratio / Void Index
6. Wave Velocity ⇒ Competency
* 7. Compressive Strength
* 8. Modulus Value
* 9. Joint spacing
* 10. Joint Inclination
* 11. Joint Surface Characterization

Most Important

Density :->

Class	Dry Density	Description
1	<1.8	V. low
2	1.8 - 2.2	low
3	2.2 - 2.55	Moderate
4	2.55 - 2.75	High
5	>2.75	V. High

Some wave Velocity :->

Class	Velocity (m/s)	Description
1	<1500	V. low -> Incompetent
2		
3		
4		
5	>5000	more competent

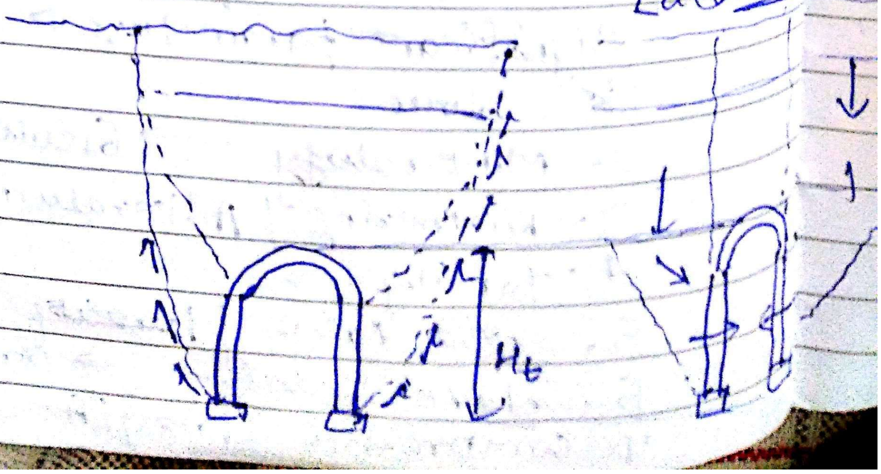
Deere - Miller.

V. high strength -> >200 MPa -> A	Mod. Ratio
High -> 100 - 200 " -> B	M_r > 500 => High (H)
medium -> 50 - 100 " -> C	200 - 500 => Medium (M)
low -> 25 - 50 " -> D	<200 => Low (L)
V. low -> <25 " -> E	

Rock Mass Classification :->

1) Terzaghi (1946) ->

"Tunels"



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Rock Mass Classification Systems

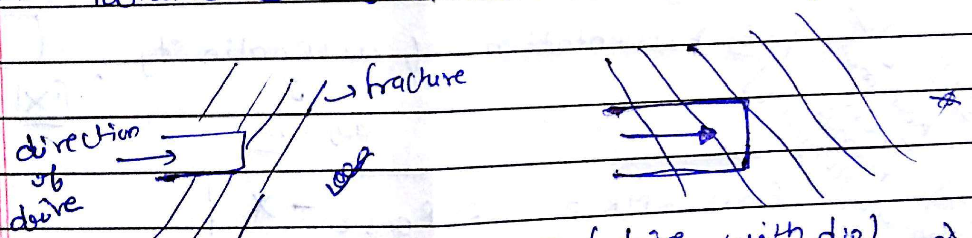
(i) Terzaghi's Road Load Classification System
(ii) RQD
(iii) Rock Structure Rating (RSR)

— Parameters :-

— Geological Parameters	Construction Constant Parameters
a) Rock Type	a) direction of drive (for tunnels)
b) Joint Pattern	b) Size of tunnel
c) Joint orientation	c) Method of excavation
d) faults/ folds	d) (Drill-Blast / TBM)
e) Rock Material Property	
f) weathering	

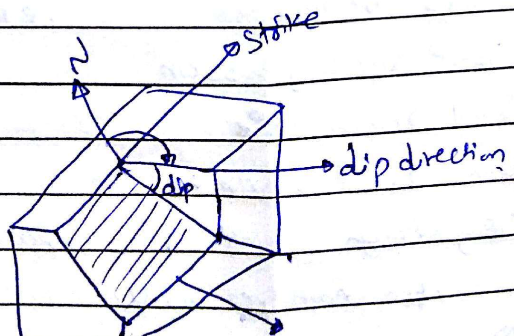
(use Table and chart)

Parameter A :- General Area Geology
Parameter B :- Joint Pattern, direction of drive



Case ① (drive against dip) Case ② (drive with dip) (adopt this)

(with the dip better case)



example dip = 40°, joint spacing = 15 cm 2-6 inch.

Driving with Dip, Strike ⊥ to Axis of tunnel

Parameter B ⇒ weightage (36 (Rating) from Tables.

Parameter C :- Ground water, joint Condition

ex $A = 24, B = 36$

$C_? = A + B = 60$

joint condition = good

500 gpm / 1000 ft of tunnel = Anticipated water flow

So, Rating = 21 (weightage)

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(iv) Rock Mass Rating (RMR) (Table 4.4)

Parameters :-

a) UCS of intact Rock → 15% max. weightage
b) RQD → 20% max. weightage
c) spacing of discontinuity → 20% " "
d) Condition of discontinuity → 30% "
e) ground water condition → 15% "

(RMR_{Basic} = a to e → 100%)

f) Orientation of discontinuity → adjustment (X)

exam // Remember These names and parameters

$$\boxed{\text{Final RMR} = RMR_{Basic} - X}$$

Example UCS = 55 MPa, RQD = 70%

Joint spacing = 33 cm

Joint Dip = 35°

Tunnel axis Parallel to strike

Very Rough Joints, no separation, not Continuous, Unweathered wall

Damp.

a = 7	d = 30	f = −5 (fair Condition for tunnels as ...)
b = 13	e = 10	
c = 10		

Final RMR = 7+13+10+30+10−5

⇒ 70−5 ⇒ 65 Ans

Date : / /

Rock Mass Classes

⇒ Good Rock [Rating ∈ (61, 80)]

Class number = II (Good Rock)

So, average stand up time = 1 Year for 10m opening span

(यानी one year तक support नहीं चाहिए Otherwise tunnel गिर जायेगी)

Cohesion of (C) Rock Mass = 300 - 400 kPa

(ϕ) = 35-45

~~In above case~~ Penalty for foundations = / ✗ 2)

(V) Rock Mass Quality Index, Q (1973).

(Q system) (most popular system)

exam Remember

$$Q = \left[\frac{RQD}{J_n}\right]\left[\frac{J_r}{J_a}\right]\left[\frac{J_w}{SRF}\right]$$

Table = 4.6

$$Q = \frac{RQD}{J_n} \times \frac{J_r}{J_a} \times \frac{J_w}{SRF}$$

Parameters

i) RQD = Rock Quality Designation (spacing)

ii) J_n = Joint set number (no. of joint set)

if J_n increase than RQD will decrease.

$$\frac{RQD}{J_n} = \text{Block Size}$$ (gives you idea about

iii) J_r = Joint Roughness number

iv) J_a = Joint Alteration number

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$\frac{J_r}{J_a}$ → gives you idea about ϕ (friction)

v) J_w = Joint water Reduction

vi) SRF = Stress Reduction factor.

Range of Q = 0.001 to 1×10^3

7/4/13 Q System:- Table 4.6

Six Parameter (Remember)

ESR = Excavation support Ratio

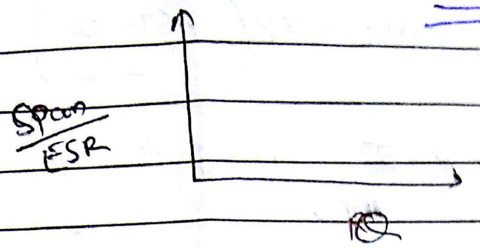
→ depending upon the importance of structure

Excavation Category	ESR
A. Temporary Mine Opening	3-5
B. Vertical Shaft	
– Circular	2.5
– Reactangle	2.0
C. Permanent Mine opening	→ 1.6
D. Storage Rooms, Water Treatment Plant, Minor Highway	→ 1.3
E. Power Station, Major Highway	→ 1.0
F. Nuclear Power Station	→ 0.8

Support Design using Q System

See Figure 4.3

Figure 4.3



$$\text{Corrected span} = \frac{\text{Span or Height in m}}{ESR}$$

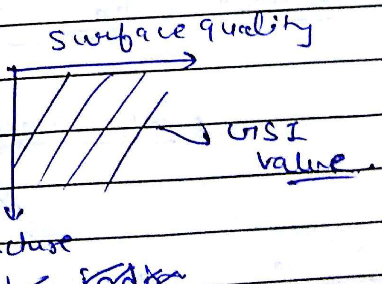
* We make the Rock Self supporting by Providing Bolts and Reinforcement

* These Empirical design is oversafe — RMR, RSR, Q

(Vi) Geological Strength Index (GSI) (Latest classification system) for blocky Jointed Rock Mass (1996) (very simple)

2 important Parameters
→ structure Type
→ surface quality

From graph we calculate GSI value.



~~This GSI value is used~~ For (Jointed Rock Mass)
Hoek Brown Criterion

$$\boxed{\sigma_1' = \sigma_3' + \sigma_{c_i}\left[m_j\left(\frac{\sigma_3}{\sigma_{c_j}}\right) + S_j\right]^a}$$

stress are effective stress

for intact Rock → $a = 0.5$, $S_j = S_i = 1$

$\boxed{\sigma_1' = \sigma - u}$

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For jointed Rock Mass

$$m_j = m_i \exp\left(\frac{GSI-100}{28}\right)$$

$GSI > 25$, $S_j = \exp\left(\frac{GSI-100}{9}\right)$; $a = 0.5$

$GSI < 25$, $S_j = 0$, $\boxed{a = 0.65 - \frac{GSI}{200}}$

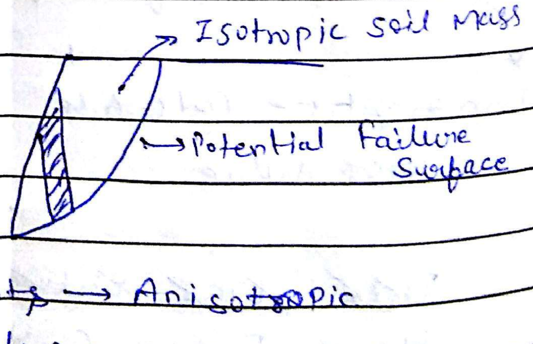
example

- fair surface quality
- structure looks ~~blocky~~

	Structure	surface	GSI	Approx GSI
1.	Blocky	fair	45-65 ✓	~50
2.	Disintegrated	Poor	15-25	~40
3.	Sheared	very Poor	0-15	~20
4.	very Blocky	Good	45-65	~50
5.	Intact or Massive	Good	70-80	~35
6.	Blocky/Disturbed.	Poor	25-35	~30

11-4-17 **Rock Slopes :-**

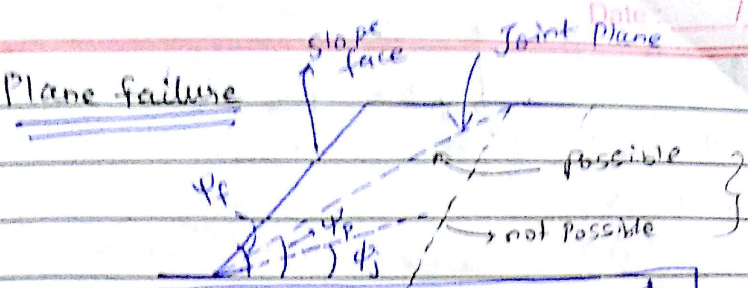
Soil Slope



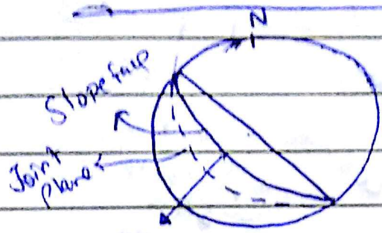
Rock slope → Joints → Anisotropic
- Planar failure
- Wedge failure
- Toppling failure
- Circular failure (heavily fractured Rock mass) (Behave as soil)

** UPCP
* If there is 2-3 sets of joints set then rock mass will behave as isotropic
* But in case of circular failure (large no. of joint sets) — anisotropic

Plane failure



$\boxed{\text{for failure } \psi_f > \psi_p > \phi_j}$



Conditions

1) Dip direction of Joint Plane } → failure is possible
dip direction of Slope face

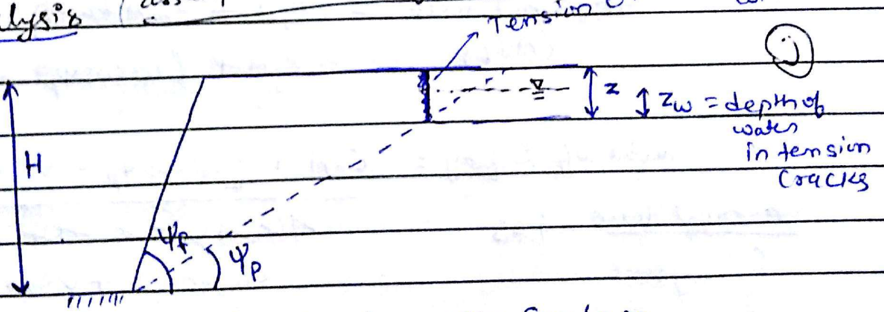
Should be in same direction

difference = ($\pm 20°$)

(??) 2) failure plane "daylights" in slope face ($\psi_f > \psi_p$)

3) $\psi_p > \phi_j$

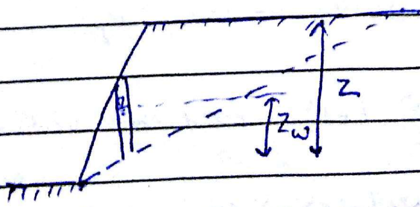
Analysis (assumption – sliding failure)



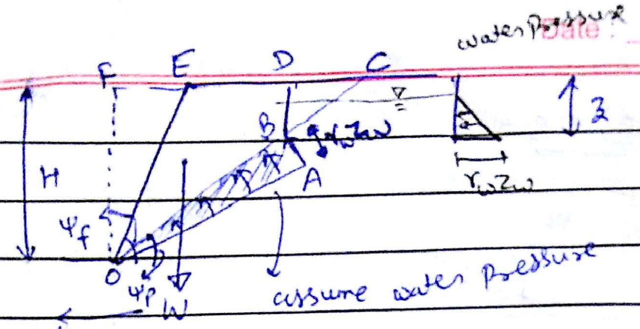
Case ① Tension Cracks in upper Surface

Case ② Tension Cracks in slope face

C



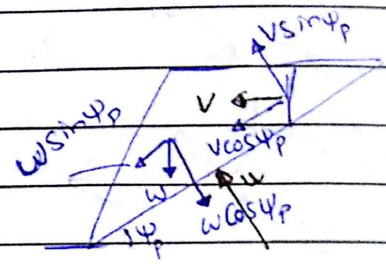
Case ①



weight of slope wedge

ar(OAB) = force per unit length on OB.

Sliding force



$\Rightarrow W\sin\psi_p + V\cos\psi_p$

normal force on potential failure surface

$\Rightarrow W\cos\psi_p - V\sin\psi_p - u$

Shear strength of Joint plane :- Coulomb's, Patton, Barton, LA

lets use Coulomb's eqn $\Rightarrow c_j, \phi_j$

frictional force $(A\tau_f)$ $= c_j A + \sigma_n A \tan\phi_j$

$= c_j A + [W\cos\psi_p - V\sin\psi_p - u]\tan\phi_j$

factor of safety $= \dfrac{c_j A + [W\cos\psi_p - V\sin\psi_p - u]\tan\phi_j}{\cancel{W\cos\psi_p - V\sin\psi_p - u} \quad W\sin\psi_p + V\cos\psi_p}$

$\dfrac{\text{frictional force}}{\text{Sliding force}} = FOS$

$$FOS = \frac{c_j A + [W\cos\psi_p - V\sin\psi_p - u]\tan\phi_j}{W\sin\psi_p + V\cos\psi_p}$$

against sliding

for case ① let's find the values

ar(OEC) = ar(OFC) − ar(OFE)

$= \frac{1}{2}\cdot H \cdot \frac{H}{\tan\psi_p} - \frac{1}{2} H \cdot \frac{H}{\tan\psi_f}$

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$ar(OEC) = \frac{H^2}{2}[\cot\psi_p - \cot\psi_f]$

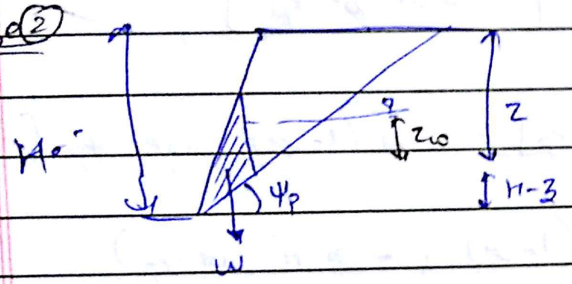
$ar(BDC) = \frac{z^2}{2}\cot\psi_p$

So, Area of wedge $[OEDA]$

$= \frac{H^2}{2}[\cot\psi_p - \cot\psi_f] - \frac{z^2}{2}\cot\psi_p$

$$\boxed{\text{So, weight of wedge } (W) = \frac{1}{2}\gamma H^2\left(1 - \frac{z^2}{h^2}\right)\cot\psi_p - \cot\psi_f}$$

For Case (2)



we can calculate weight of shaded part.

Tension Crack :- (depth of tension crack)

i) either from field

ii) Theoritical

Critical Tension Crack (for Dry Condition)

for Dry Condition FOS $(u=0, V=0)$

$$FOS = \frac{c_j A + (W\cos\psi_p)\tan\phi_j}{W\sin\psi_p}$$

-u] $\tan\phi_j$

$\tan\phi_j$

$$F = \frac{c_j A}{W\sin\psi_p} + \cot\psi_p \tan\phi_j \quad [\text{for Dry Condition}]$$

for Critical Tension Crack :- Say $\frac{z}{H} = x$

$F = f(A, W)$

for minimum Tension ~~crack~~ FOS.

$\frac{\partial F}{\partial x} = 0 \Rightarrow \cancel{\frac{c_j \frac{\partial A}{\partial x} - \sin\psi_p \frac{\partial W}{\partial x}}{(W\sin\psi_p)^2} = 0}$

$$\frac{\partial F}{\partial x} = \frac{c_j}{\sin\psi_p}\left[W\frac{\partial A}{\partial x} - A\frac{\partial W}{\partial x}\right] = 0$$

UPCP

$$w \frac{\partial A}{\partial r} = A \frac{\partial w}{\partial r}$$

Homework

[Derive and get r

$$w = \frac{1}{2}\gamma H^2 \left[(1-r^2)\cot\psi_p - \cot\psi_f \right]$$

$$A = (H-Z)\,\operatorname{cosec}\psi_p$$

$$A = H\{1-r\}\operatorname{cosec}\psi_p$$

$$\frac{\partial A}{\partial r} = H(-\operatorname{cosec}\psi_p)$$

$$\frac{\partial w}{\partial r} = \frac{1}{2}\gamma H^2 \left[-2r\cot\psi_p \right]$$

$$\Rightarrow \quad w\cancel{H}[-\cancel{\operatorname{cosec}\psi_p}] = \cancel{H}(1-r)\,\cancel{\operatorname{cosec}\psi_p}\left[\frac{1}{2}\gamma H^2(-2r\cot\psi_p)\right]$$

$$= -w = (1-r)(-r^2H^2\cot\psi_p)$$

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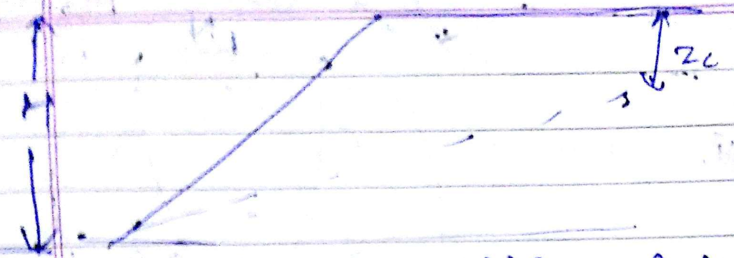


$$\frac{W\,\partial A}{\partial r} = A\frac{\partial W}{\partial r}, \quad \boxed{r = \frac{z_c}{H}}$$

$$W = \frac{1}{2}\gamma H^2\left[(1-r^2)\cot\psi_p - \cot\psi_f\right]$$

$$A = H(1-r)\,\mathrm{cosec}\,\psi_p, \quad \frac{\partial A}{\partial r} = H(-1).\,\mathrm{cosec}\,\psi_p.$$

$$\frac{\partial W}{\partial r} = \frac{1}{2}\gamma H^2\{-2r\}\cot\psi_p$$

$$r^2 - 2r + 1 - \cot\psi_f \cdot \tan\psi_p = 0$$

$$r = \frac{2 \pm \sqrt{4 - 4\times 1\times(1-\cot\psi_f\tan\psi_p)}}{2}$$

$$r = 1 - \sqrt{\cot\psi_f\tan\psi_p}$$

$$\boxed{\frac{z_c}{H} = 1 - \sqrt{\cot\psi_f\tan\psi_p}}$$

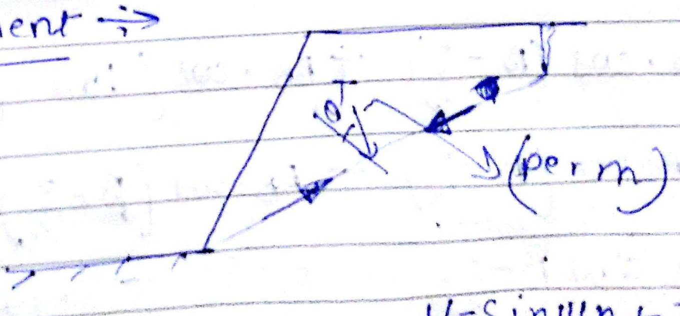
Position of tension crack ⇒

$$b = H\cdot\cot\psi_p - H\cot\psi_f - z_c\cdot\cot\psi_p$$

$$\frac{b}{H} = \left(1 - \frac{z_c}{H}\right)\cot\psi_p - \cot\psi_f$$

$$\frac{b_c}{H} = \sqrt{\bigcirc} - \cot\psi_f$$

Reinforcement ⇒



$$FOS = \frac{cA + (W\cos\psi_p - U - V\sin\psi_p + T\cos\theta)\tan\phi}{W\sin\psi_p + V\cos\psi_p - T\sin\theta}$$

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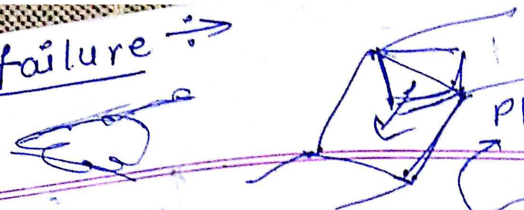
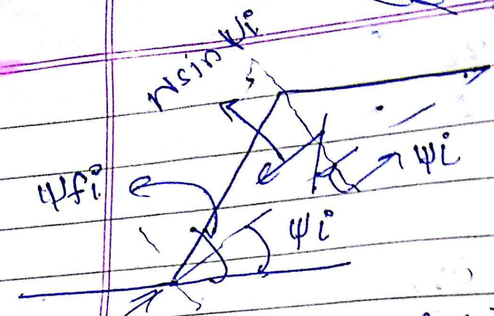
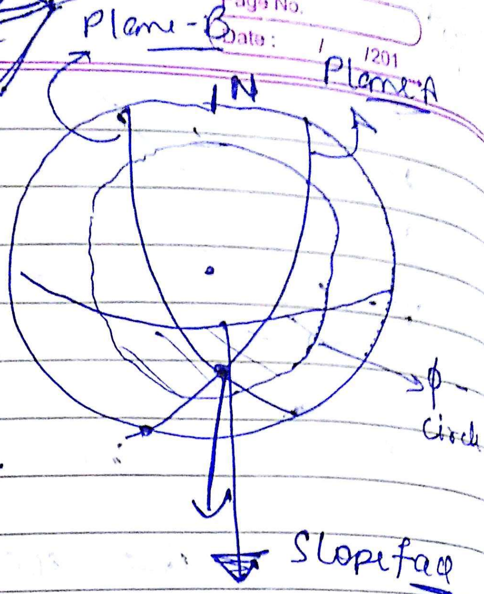
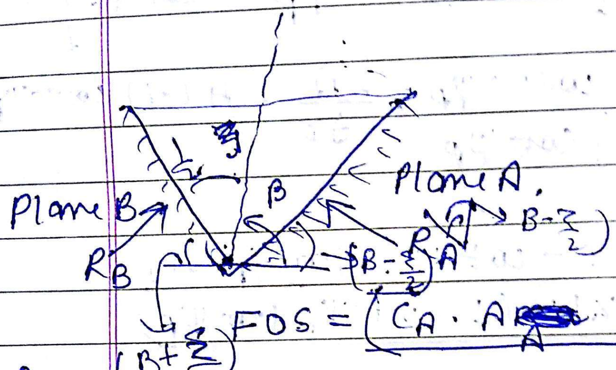
Wedge failure :-







ψ_i = Plunge of line of intersection.



Plane A → flatter
" B → steeper

$$FOS = \frac{(C_A \cdot A_A + R_A \tan\phi_A) + (C_B A_B + R_B \tan\phi_B)}{N \sin\psi_i}$$

$$C_A = 0,\quad C_B = 0,\quad \phi_A = \phi_B = \phi$$

$$FOS = \frac{(R_A + R_B)\tan\phi}{N \cdot \sin\psi_i}$$

To find R_A, R_B :-

$$R_A \cdot \sin\left(\beta - \frac{\xi}{2}\right) = R_B \cdot \sin\left\{180 - \left(\beta + \frac{\xi}{2}\right)\right\}$$

$$\boxed{R_A \cdot \sin\left(\beta - \frac{\xi}{2}\right) = R_B \cdot \sin\left(\beta + \frac{\xi}{2}\right)} \quad \text{—— (1)}$$

$$R_A \cdot \cos\left(\beta - \frac{\xi}{2}\right) + R_B \cdot \cos\left\{180 - \left(\beta + \frac{\xi}{2}\right)\right\} = W\cos\psi_i$$

$$R_A \cdot \cos\left(\beta - \frac{\xi}{2}\right) - R_B \cdot \cos\left(\beta + \frac{\xi}{2}\right) = W\cos\psi_i$$

$$R_B\left[\sin\left(\beta + \frac{\xi}{2}\right)\cot\left(\beta - \frac{\xi}{2}\right) - \cos\left(\beta + \frac{\xi}{2}\right)\right] = W \cdot \cos\psi_i$$

↓ show

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$$\boxed{R_B = \frac{W \cos\psi_i \cdot \sin\left(\beta - \frac{\xi}{2}\right)}{\sin \xi}}$$

$$\boxed{FOS = \frac{\sin\beta}{\sin\frac{\xi}{2}} \cdot \frac{\tan\phi}{\tan\psi_i}}$$