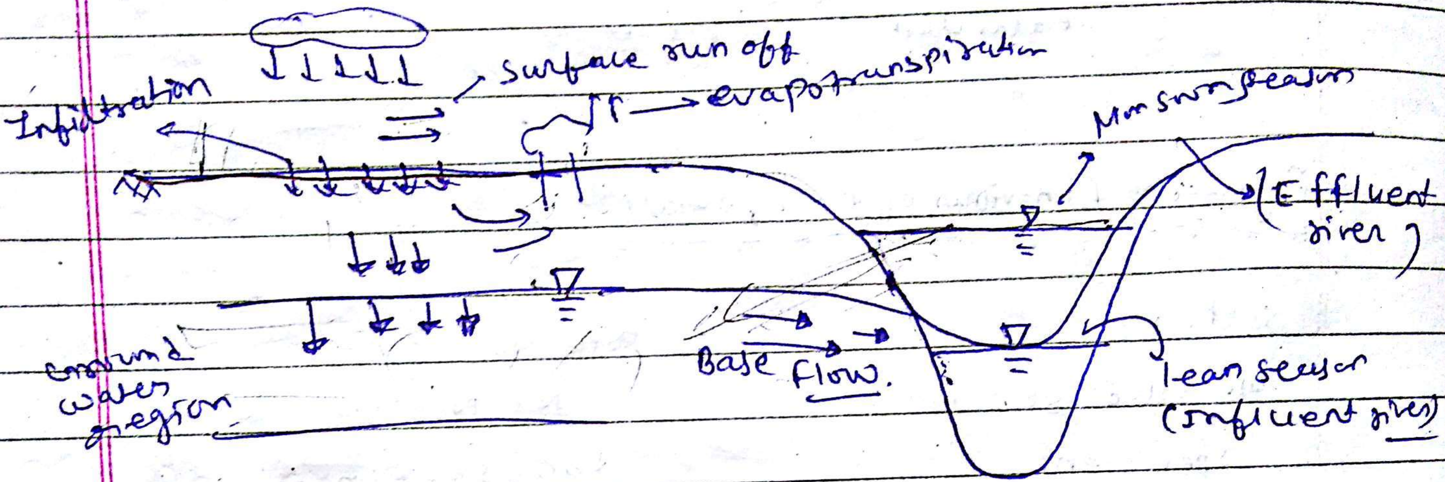
Ground water Hyd engineering

Books (1) Ground water - Freeze and Cherry

(2) Ground water - H M Raghunath

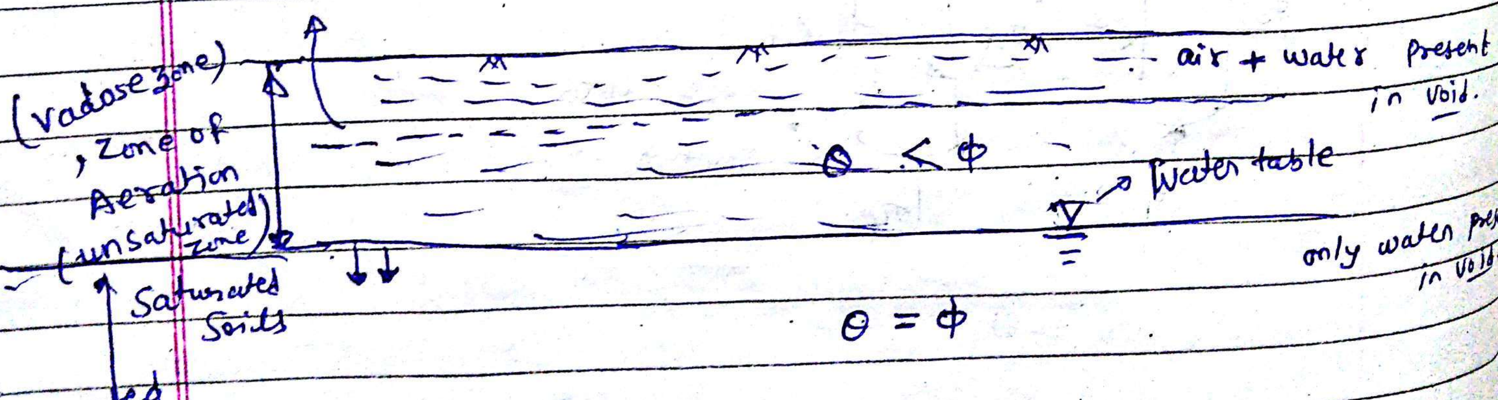
Ground water



during Monsoon season, river water level is higher than ground water.

so, river to groundwater

Sub surface water = water present below ground surface



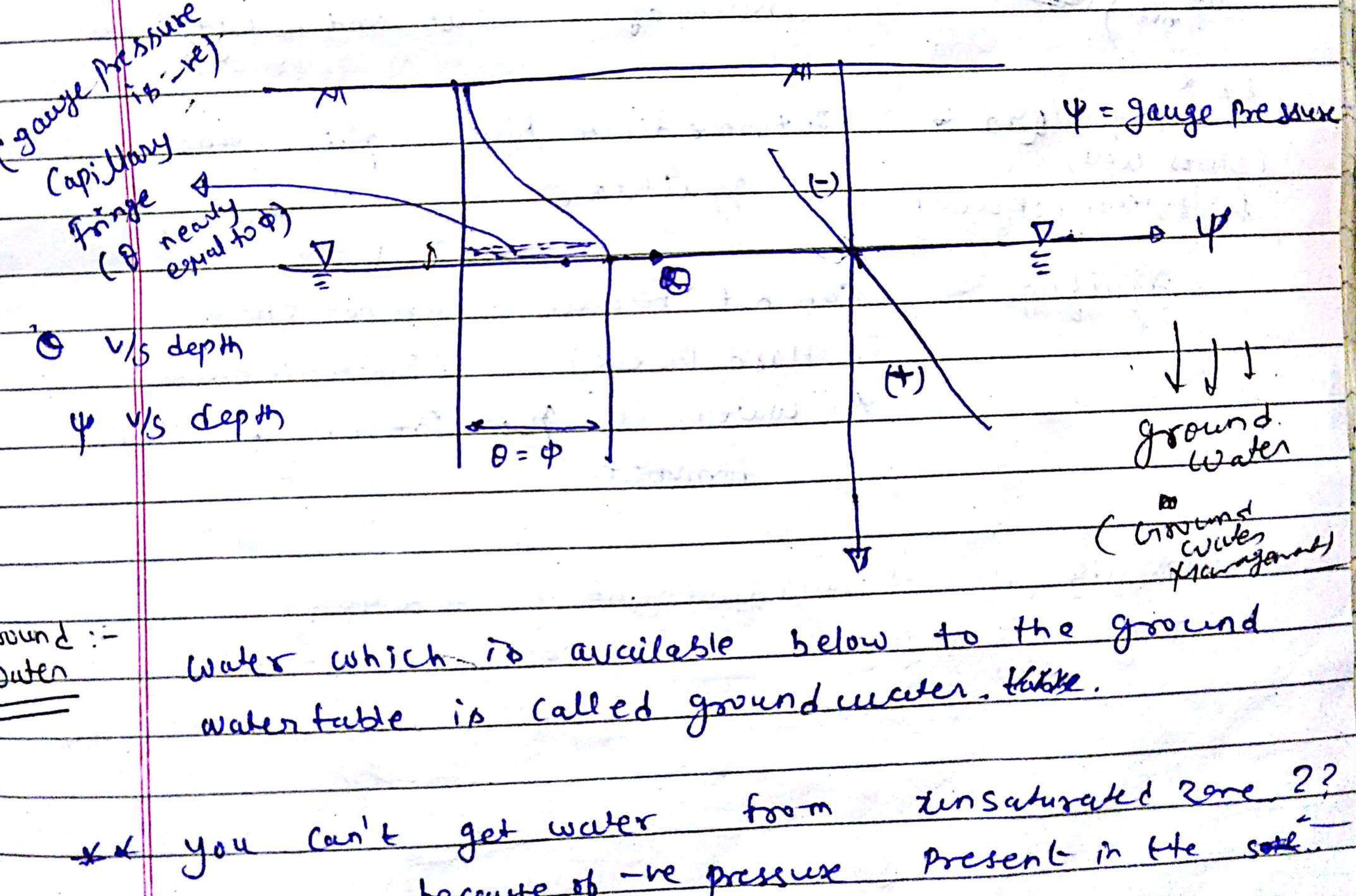
Moisture content $= \theta$ (in terms of volume in Hydrology)

$$\Rightarrow \frac{\text{Volume of Water}}{\text{Volume of Soil}}$$

Porosity (ϕ) $= \frac{V_v}{V_T}$ = $\frac{\text{Volume of Pores}}{\text{Volume of soil}}$

$\theta \leq \phi$

* Surface which differentiate b/w. Zone of aeration and saturation is called water table, subjected to atmospheric pressure, is also called phreatic surface.



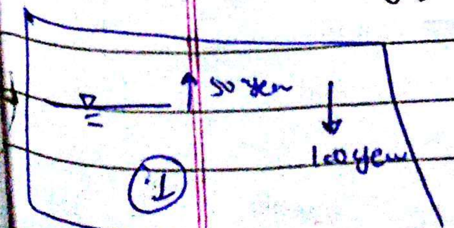
θ v/s depth

ψ v/s depth

Ground water :- Water which is available below to the ground watertable is called groundwater. ~~table~~

** you can't get water from unsaturated zone ?? because of -ve pressure present in the ~~soil~~ water

Water logging :- rise of water table to root zone.



→ Zone of unsaturated soil is important for irrigation and agricultural engineer

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Date
Page

* Irrigation schedules.

Aquifer:- Saturated a Geological formation from which it is easy to extract water & water from there.

(can store very well and release water very well)

for example: gravel formation
Sand formation.

Aquiclude:- - Difficult to take water from it.
- Saturated Geological formation
consist of silt and clay.

(Store very well, but release not)

** **Aquitard :-** Intermediate b/w aquifer and aquiclude
aquitard.

(stores well, but release not well)

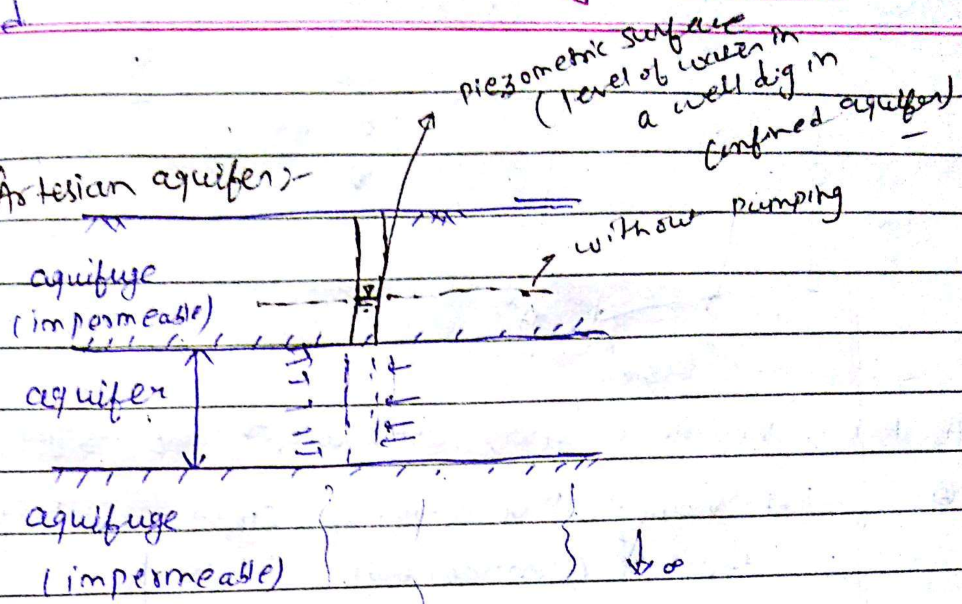
Aquifuge:- Can not release, can not store
(Hard Rock) (Impermeable)
* water is not Present in this formation

Porosity (η) of sand and gravel => 30 to 40%
Clay = 50 to 60% [Per unit volume of clay contain more water than sand and gravel]

	Store	Release
Aquifer	✓	✓
Aquiclude	✓	✗
Aquitard	✓	✓✗
Aquifuge	✗	✗ ← Impermeable rock

Aquifer ⟨ Confined

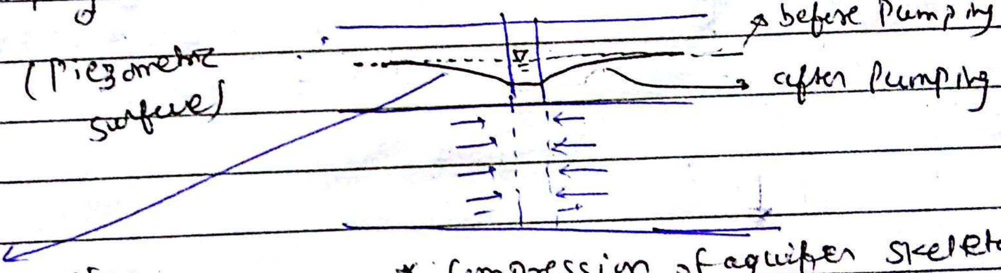
① Confined aquifer / Artesian aquifer :-



aquifer
* Confined b/w two aquifuge or two impermeable layer is called Confined aquifer.

* level up to which water will rise when you dig a well is called Piezometric Surface.

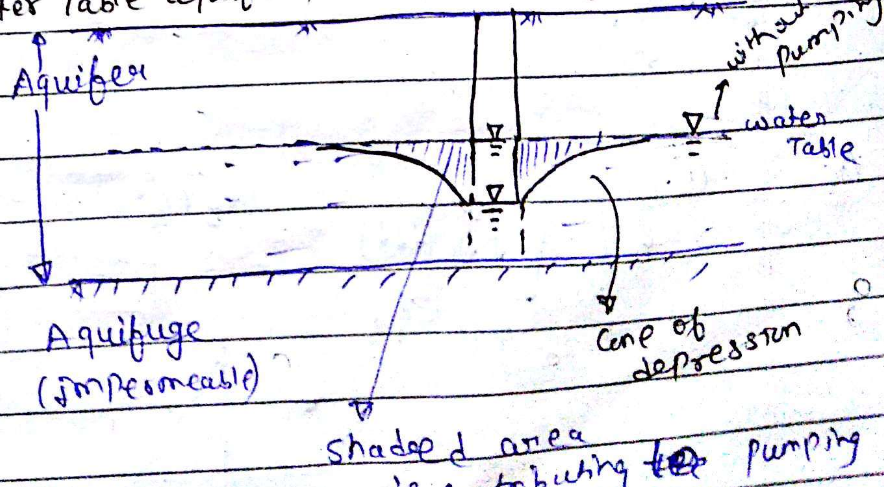
* after Pumping



Cone of depression

* Compression of aquifer skeleton.

② Unconfined aquifer / Water Table aquifer / Phreatic aquifer



* starts from Top.
* only underlain by aquifuge.

Shaded area is contributing the pumping

* Water is coming through gravity drainage of Pores of soil.

* Shaded area से ESI extract करेंगे

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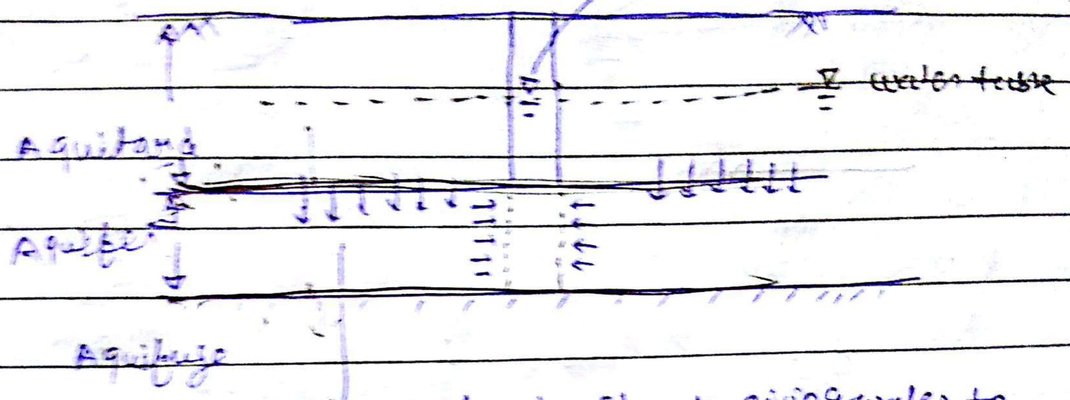
③ Leaky Aquifers

overlain by aquitard

underlain by aquifuge



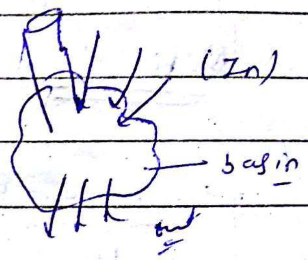
aquitard starts giving water to aquifer (This process is called leakage) i.e. after pumping.

Ground Water Hydrology

* In the zone of saturated zone, we use piezometer for pressure calculation
* In the zone of unsaturated zone, we use tensiometer (-ve pressure)
  → water enters from Tensometer to dry (unsaturated) soil such that soil become saturated.
  → more the amount of water it takes, more -ve pressure is there.

Components of (Input and output of ground water)

1. Recharge of ground water { Rainfall, Irrigation }



2. Ground water flow going out of the basin
3. Ground water flow to stream adjoining rivers or stream.
4. pumping in the ground water
5. Change in storage

Ground water Balance

(1) − [(2) + (3) + (4)] = Change in storage = (5)

(1) → depends on aquifer characteristics → Generally depends on (Rainfall)

(2) → depends; (3) → depends → aquifer characteristics

* Characteristics of aquifer is required

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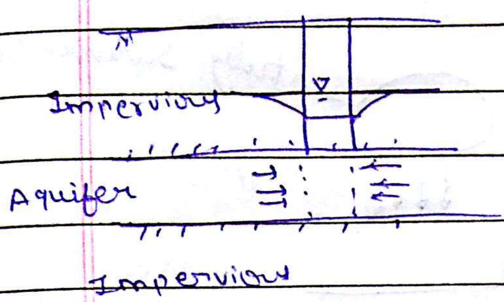
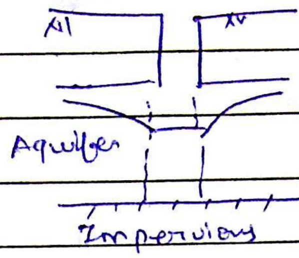
Ground water Hydrology

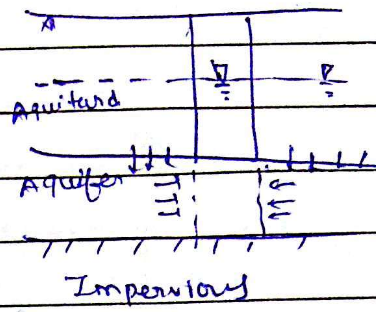
Confined Aquifer



unconfined aquifer

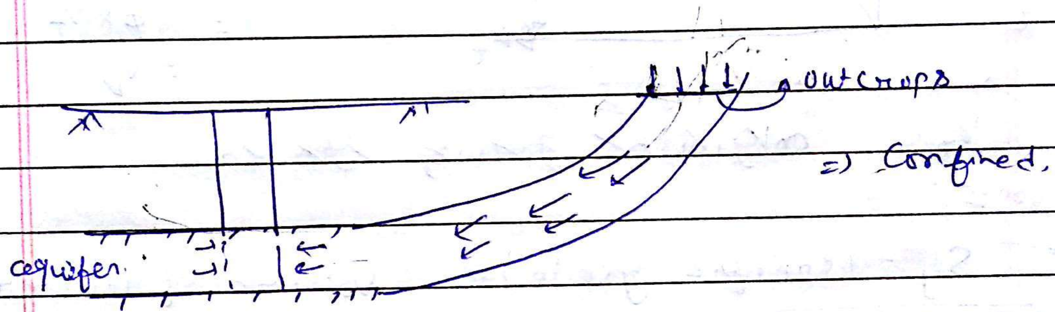


Leaky aquifer



Source of water in aquifer:-

1. → Rainfall — in unconfined
2. → Stream aquifer interaction → in unconfined



* Confined gets recharge from outcrops. expose to ground surface.
* in Real life we mostly encountered with Leaky aquifer.
* If aquitard thickness in leaky aquifer is quite large, it acts as confined aquifer.
* If leaky aquifer ~~is~~ aquitard thickness is small than it acts as unconfined aquifer.

* Quantity points of view = leaky aquifer
* Quality point of view ⇒ Confined aquifer.

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* If water table is deep than, quality of water is good.

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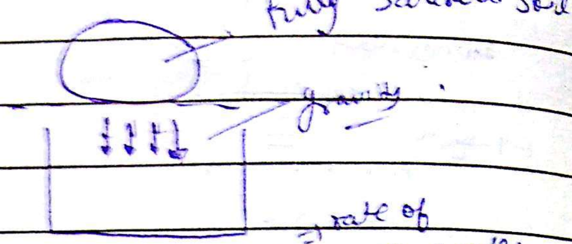
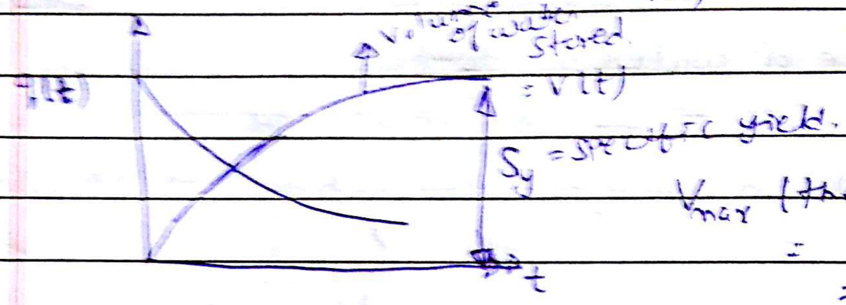
1) Storage Characteristics
2) Flow characteristics



$q(t)$ – ~~volume~~ rate of volume of water (m³/s)



V_{max} (theoritical) $= \phi \times V_T$ $= V_V$

~~Vmax achieving gravity due to~~

percentage volume

S_y = (specific yield) ⇒ $\dfrac{\text{Volume of water drained by gravity}}{\text{Total Volume of soil}}$

$\boxed{S_y \le \phi}$

$\boxed{S_y V_T \le V_{max}}$

Boulton's Experiments:- (for unconfined aquifer)

$q(t) = Ae^{-\alpha t}$ A, α ⇒ depends on soil Characteristic ⇒ soil parameter.

percentage volume

$\int_0^\infty q(t)\,dt = S_y$

$\Rightarrow \int_0^\infty Ae^{-\alpha t}\,dt = S_y$

$\Rightarrow \left[\dfrac{Ae^{-\alpha t}}{-\alpha}\right]_0^\infty = S_y$

$\Rightarrow \dfrac{-A}{\alpha}[0-1] = S_y$

$\boxed{A = \alpha S_y}$

So, $q(t) = \alpha S_y e^{-\alpha t}$

$$\boxed{q(t) = \alpha S_y e^{-\alpha t}}$$

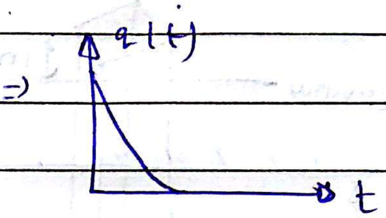
S_y = specific yield = ultimate vol. of water that can be released from unit volume of soil by gravity.

$\left(\frac{1}{\alpha}\right) \Rightarrow$ delayed index

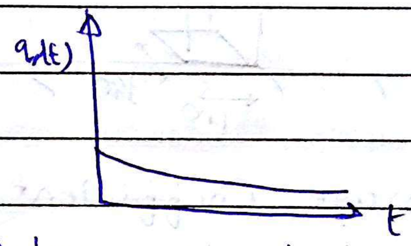
$\left(\frac{1}{\alpha}\right) \Rightarrow$ unit is "second"

If α is very High

$q(t) = \alpha S_y e^{-\alpha t} \Rightarrow$



If α is very low



For gravels delayed index = in minutes

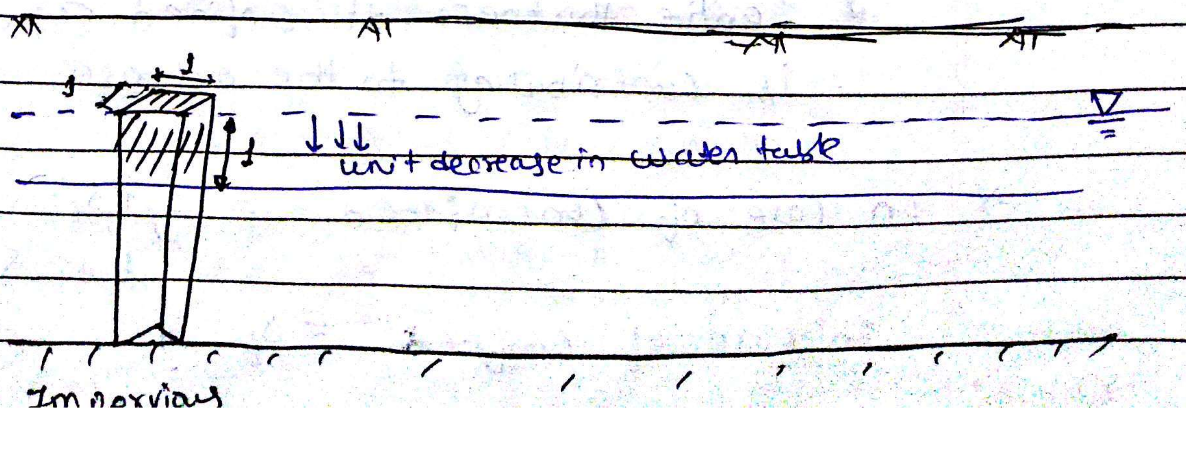
For Sand delayed index $\Rightarrow$ in Hours.

For clays delayed index $\Rightarrow$ in days.

S_r (specific Retention) = volume of water which is retained after gravity drainage

$$\boxed{\boxed{S_y + S_r = \phi}} \Rightarrow ✓$$

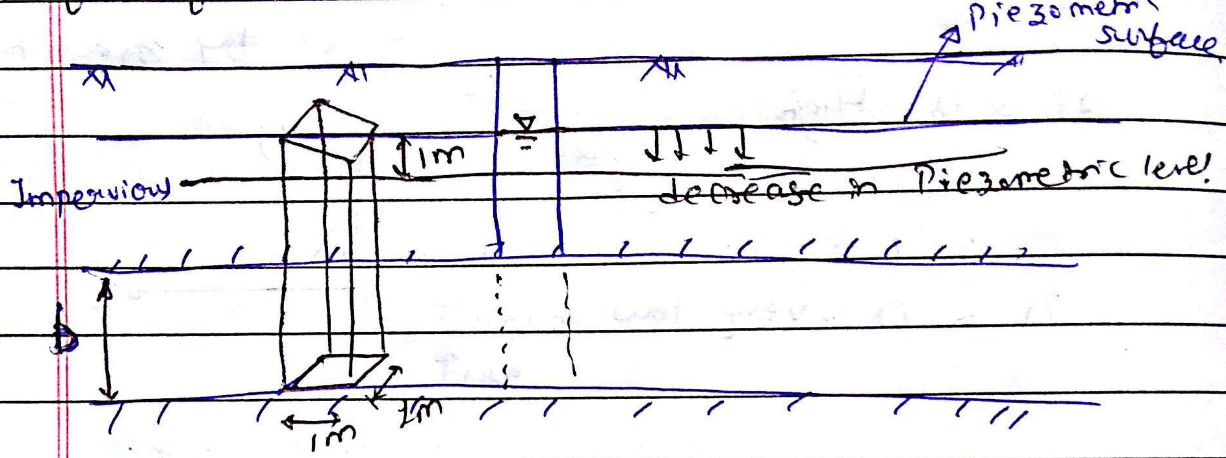
unconfined



$S_y \Rightarrow$ Volume of water released per unit area of aquifer per unit decline in water table elevation.

* It is unit less quantity.

In Case of Confined



Storage Coefficient (S_t) = Volume of Water Released from a confined aquifer per unit area of aquifer, per unit decline in Piezometric surface.

Total stress (σ) = $\bar{\sigma} + \mu$ (pwp) at aquifer.

after pumping μ will decrease
so, $\bar{\sigma}$ will increase.

So, aquifer gets will compress.

Bez of that water will release.

* entire thickness of confined aquifer is contributing to the release.

* In Case of Unconfined = $S_y + S_t$

but $S_t \ll S_y$

In Case of Confined = S_t

(S_s) Specific Storage (for confined aquifer):-
The volume of water release from a confined aquifer per unit decline in Piezometric surface per unit volume of aquifer.

$$\boxed{S_t = b\,S_s}$$

$S_s \Rightarrow$ Property of aquifer
(For same aquifer, S_s will be same)

So, for unconfined aquifer $\Rightarrow$ 3 Parameters
S_y, $1/\alpha$, $S_t \longrightarrow$ negligible

For confined aquifer $\rightarrow$ S_t = Storage coefficient
$\hookrightarrow 10^{-6}$ to 10^{-3}

Flow characterstics:-
Permeability Hydraulic Conductivity

$$Q = KiA$$

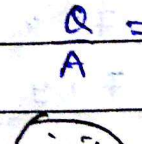
Darcy's law:-

$$Q \propto A \;;\; Q \propto (H_1 - H_2) \;;\; Q \propto 1/L$$

So, $Q = K \dfrac{(H_1 - H_2)}{L} A$ $i = \dfrac{H_1 - H_2}{L}$

So, $\boxed{Q = KiA}$ i = Hydraulic gradient

$\dfrac{Q}{A} = Ki = V$ = darcy's velocity



Actual flow area = $A \times \phi$ $\rightarrow$ Porosity

$$A_f = A\phi$$

$$V_s = \frac{Q}{A_f} = \frac{Q}{A\phi} = \frac{V}{\phi}$$

$V_s = \dfrac{V}{\phi}$ $V \rightarrow$ darcy's Velocity

$\swarrow$

Actual velocity = Seepage velocity

$$\boxed{V_s = \frac{V}{\phi}}$$

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Darcy's law is valid only for Laminar flow

$$R_e = \frac{\rho V d}{\mu} = \frac{Vd}{\eta}$$

$V \rightarrow$ darcy's velocity

$d \rightarrow d_{50}$

$\eta \rightarrow$ Kinematic viscosity

$$R_e = \frac{V d_{50}}{\eta}$$

In Ground Water

$R_e < 1$, — Laminar Flow

$R_e > 10$ — Turbulent flow

In Channel water (Pipe)

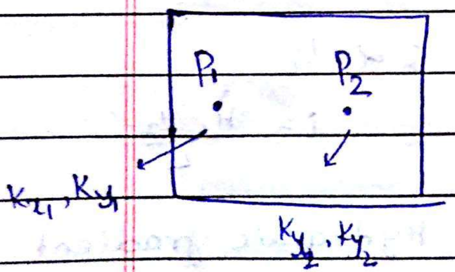
$R_e < 2000$ Laminar Flow

$R_e > 4000$ Turbulent Flow

Ground water Medium is Anisotropic in nature:-

$K_x, K_y, K_z \longrightarrow$ Anisotropic

(different direction have different Hydraulic Conductivity)



Isotropic Homogenous

$K_{x_1} = K_{x_2}$

$K_{y_1} = K_{y_2}$] $\rightarrow$ All four quantities are equal

Isotropic not Homogenous

$K_{x_1} = K_{y_1}$; $K_{x_1} \neq K_{x_2}$

$K_{x_2} = K_{y_2}$; $K_{y_1} \neq K_{y_2}$

Anisotropic Homogenous

$K_{x_1} \neq K_{y_1}$; $K_{x_1} = K_{x_2}$

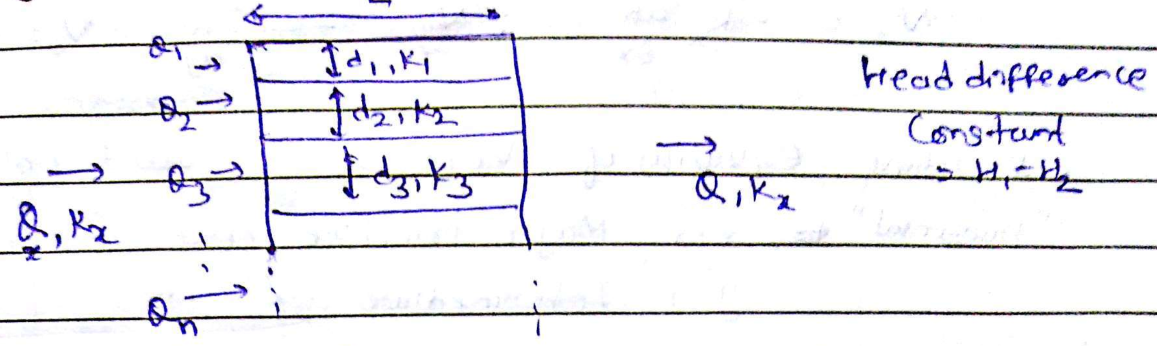
$K_{x_2} \neq K_{y_2}$; $K_{y_1} = K_{y_2}$

Anisotropic Non Homogenous

$K_{x_1} \neq K_{x_2} \neq K_{y_1} \neq K_{y_2}$

* Homogenous :- Same property in same direction
* An Isotropic :- Same property at a point in all direction.

* Soil is Anisotropic in nature having different Hydraulic Conductivity in different direction

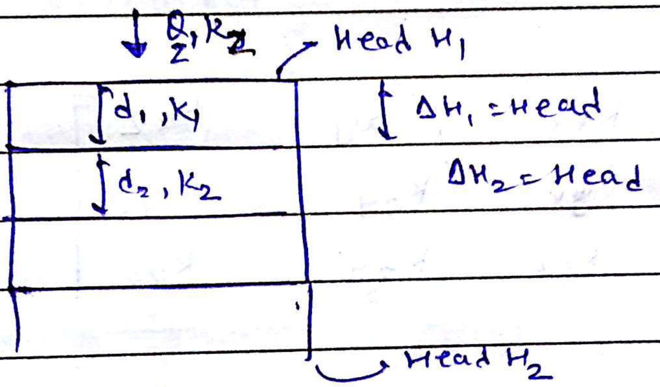


$$Q_x = \sum Q_i$$

$$\Rightarrow K_x \frac{(H_1 - H_2)}{L} (\sum d_i) \times 1 = \sum K_i \frac{(H_1 - H_2)}{L} d_i \times 1$$

$$\boxed{K_x = \frac{\sum K_i d_i}{\sum d_i}}$$

weighted Average Mean



$H_1 - H_2 = \Delta H = \sum \Delta H_i$

Q, K_y

$$\boxed{K_y = \frac{\sum d_i}{\sum d_i / K_i}}$$

weighted Harmonic mean

$$K_x > K_z$$

$$K_z \frac{\Delta H}{\sum d_i} = Q_z = \frac{K_1 \Delta H_1}{d_1} = \frac{K_2 \Delta H_2}{d_2} = \cdots = \frac{K_n \Delta H_n}{d_n}$$

$$\Delta H = \frac{Q_z \sum d_i}{K_z} = \sum \frac{d_i}{K_i} Q_z$$

$$\boxed{K_z = \frac{\sum d_i}{\sum d_i / K_i}}$$

$v = -K \frac{dh}{dl}$

$V_x = -K_x \frac{\partial h}{\partial x}$; $V_y = -K_y \frac{\partial h}{\partial y}$, $V_z = -K_z \frac{\partial h}{\partial z}$

* above equation of V_x, V_y, V_z is valid only when ??

"horizontal" → x ⇒ Major Principle axis of anisotropy

y ⇒ Intermediate " "

"usually vertical direction" ← z ⇒ Minor "

→ only when, x, y, z direction coincide with Principle axis of anisotropy.

Hydraulic Conductivity ~~Matrix~~ Tensor =
$$\begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}$$

So,
$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} \begin{bmatrix} \partial h/\partial x \\ \partial h/\partial y \\ \partial h/\partial z \end{bmatrix}$$

arbitrary x, y, z direction in velocity

$$V_x = -K_{xx} \frac{\partial h}{\partial x} - K_{xy} \frac{\partial h}{\partial y} - K_{xz} \frac{\partial h}{\partial z}$$

$$V_y = -K_{yx} \frac{\partial h}{\partial x} - K_{yy} \frac{\partial h}{\partial y} - K_{yz} \frac{\partial h}{\partial z}$$

$$V_z = -K_{zx} \frac{\partial h}{\partial x} - K_{zy} \frac{\partial h}{\partial y} - K_{zz} \frac{\partial h}{\partial z}$$

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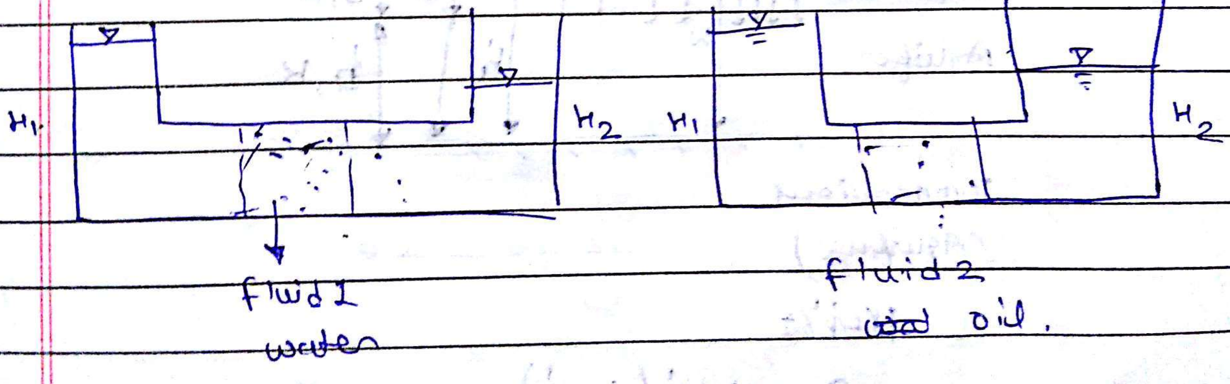
* If x, y, z direction coincide with principle axis of anisotropy then,

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = - \begin{bmatrix} k_{xx} & 0 & 0 \\ 0 & k_{yy} & 0 \\ 0 & 0 & k_{zz} \end{bmatrix} \begin{bmatrix} \partial h/\partial x \\ \partial h/\partial y \\ \partial h/\partial z \end{bmatrix}$$

* In most of the Problem we will assume that x, y, z direction is coincide with Principle axis of anisotropy..

'K' is not only depend on soil property, but also depend on the which fluid is moving.

<u>lets consider an example:-</u>



If "K" depends on only soil"
$q = K \frac{(H_1 - H_2) A}{L}$ | $q = \frac{K (H_1 - H_2) A}{L}$

But actually depends on 'fluid property
so, K depends on also fluid property.

* K depends on porous medium and fluid.

$K \propto \frac{1}{\mu}$ → viscosity

$K \propto \gamma_w$ → fluid.

So, $K = k\boxed{\frac{\gamma}{\mu}}$

K ↓ Hydraulic Conductivity

k ↓ Intrinsic Permeability of Soil

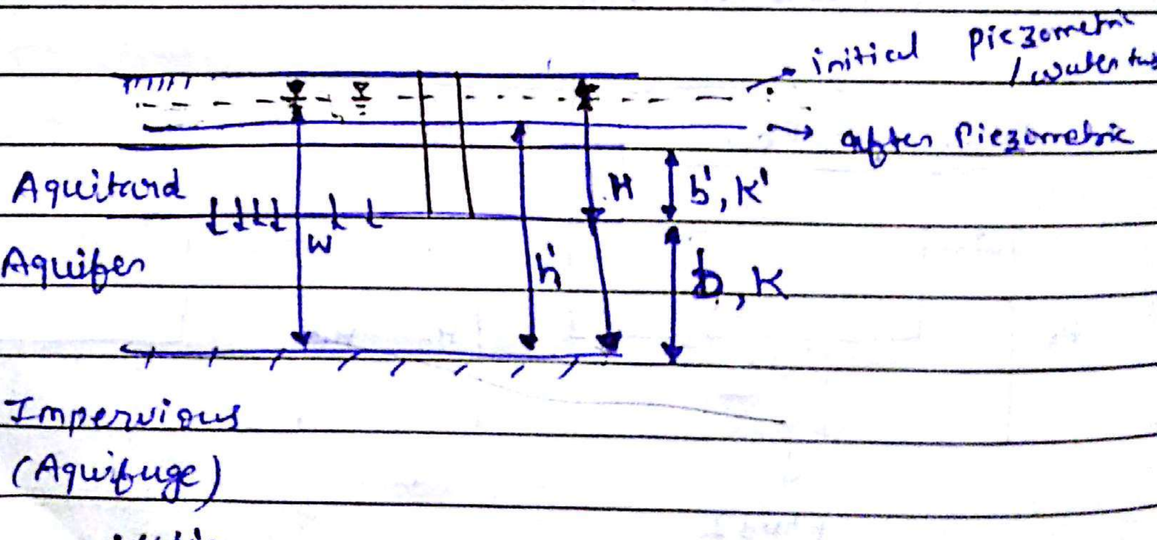
For Confined aquifer

S_t (specific storage coefficient), K

For Unconfined aquifer

S_t, S_y (specific yield), K

<u>Leaky Aquifer:-</u>



~~H-H/L~~

$$q = +K' \frac{(H-h')}{b'}$$

Per unit area

$$q = + \frac{(H-h')}{c'}$$

where

$c' = \frac{b'}{K'}$ = aquitard Resistance

If c' is very High, then it will work as Confined aquifer.

leakance (L) = $\sqrt{Kbc'}$

K = Hydraulic conductivity of aquifer

b = Aquifer thickness

c' = Aquitard Resistance

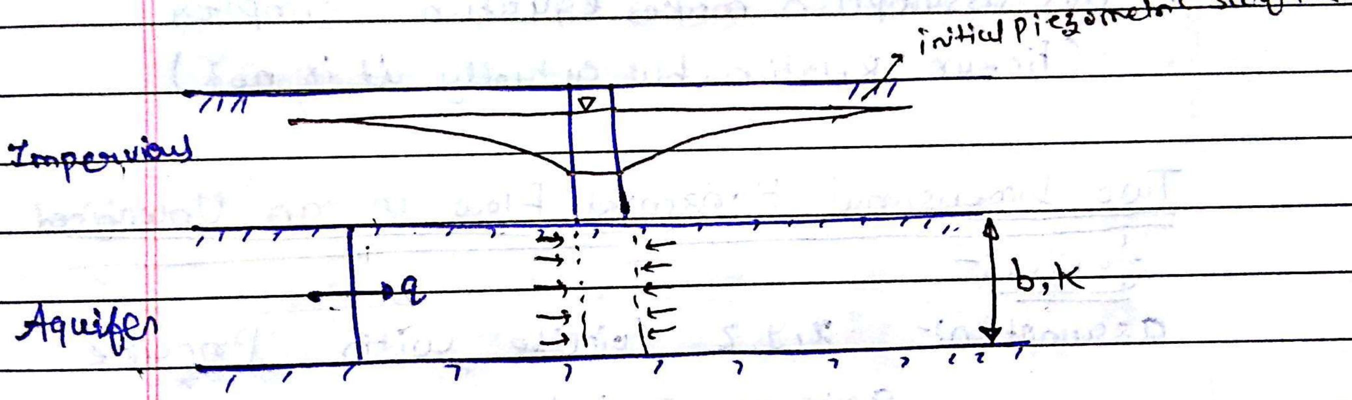
unit of 'L' is 'meter'

If L is very High it acts as confined aquifer.

If $L > 2000\,m$ — confined

$L < 500\,m$ — unconfined

Transmissivity:-

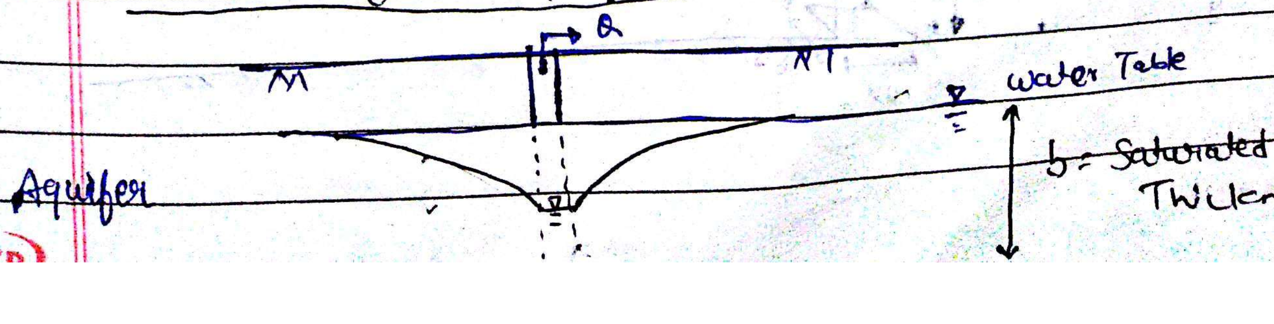


$$q = -K \frac{\partial h}{\partial l} (b \times 1)$$

$$q = -Kb \frac{\partial h}{\partial l}$$

Kb = Transmissivity (m^2/s) or (m^2/day)

For unconfined Aquifer:-



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'b' is changing so it is difficult to define Transmissivity in case of unconfined aquifer.

If drop in water table relative to saturated thickness is very less than we can define Transmissivity.

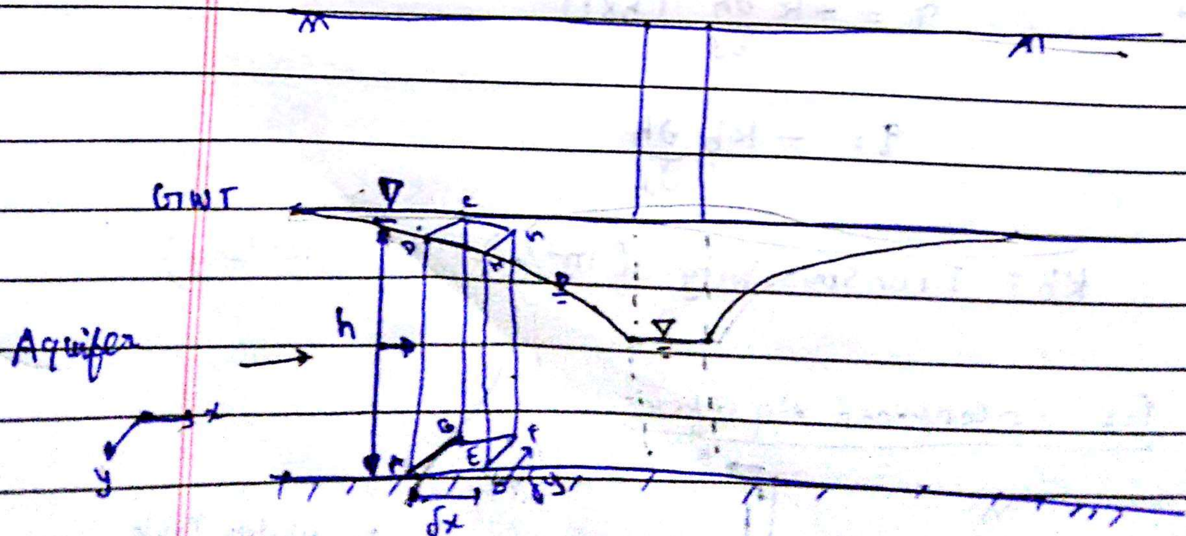
initial + final / 2 ⇒ let b' is average thickness of aquifer over a Period of Time or pumping.

so, $\boxed{T' = b'K}$ = Average Transmissivity over Period of time

This assumption makes equation simpler (Linear Relation, but actually it is not)

Two Dimensional Horizontal Flow in an Unconfined Aquifer:-

assumption:- x, y, z coincide with Principle axis of anisotropy.



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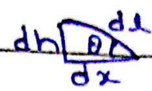
assumption

(1) → water table slope is very small (< 5°)

$$\frac{dh}{dl} \approx \frac{dh}{dx}$$



(Dupuit-Forchhiemer assumption)

- vertical component is neglected.

(2) → velocity is constant throughout the thickness at a particular time and is equal to the velocity at the water table.

$$q_i = -K_{xx}\frac{\partial h}{\partial x}(\delta y)(h) \quad = \text{Inflow Rate in } x \text{ direction}$$

$$q_o = -K_{xx}\frac{\partial h}{\partial x}(\delta y\, h) + \frac{\partial}{\partial x}\left(-K_{xx}\frac{\partial h}{\partial x}\delta y h\right)\delta x$$

(Inflow - outflow) in x direction = Storage

$$\Rightarrow (q_i - q_o)_x = \frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\delta y\, h\right)\delta x$$

$$(q_i - q_o)_y = \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\delta x\, h\right)\delta y$$

$$\Rightarrow (q_i - q_o)_x = \frac{\partial}{\partial x}\left(K_{xx}h\frac{\partial h}{\partial x}\right)\delta x\delta y$$

$$(q_i - q_o)_y = \frac{\partial}{\partial y}\left(K_{yy}h\frac{\partial h}{\partial y}\right)\delta x\delta y$$

Total Net [Inflow - outflow] = Change in storage

$$\left[\frac{\partial}{\partial x}\left(K_{xx}h\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}h\frac{\partial h}{\partial y}\right)\right]\delta x\delta y$$

$$= (S_t + S_y)\frac{\partial h}{\partial t}\delta x\delta y$$

S_y is instantaneous Release (assumption)

$S_t << S_y$

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So, $\frac{\partial}{\partial x}\left[K_{xx} h \frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[K_{yy} h \frac{\partial h}{\partial y}\right] = S_y \frac{\partial h}{\partial t}$

↓

Bousinesq equation for unconfined flow

→ h is depended variable

→ Above Bousinesq Equation is second order differential equation in space and 1st order differential equation in time.

Linearization of Bousinesq Equation:-

for unconfined aquifer $S_t \simeq 0$

$$\frac{\partial}{\partial x}\left[K_{xx} h \frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[K_{yy} h \frac{\partial h}{\partial y}\right] = S_y \frac{\partial h}{\partial t}$$

$\bar{h}$ = average saturated thickness over the time period of analysis

So,

$$K_{xx} \bar{h} \frac{\partial^2 h}{\partial x^2} + K_{yy} \bar{h} \frac{\partial^2 h}{\partial y^2} = S_y \frac{\partial h}{\partial t}$$

$$\Rightarrow \bar{T}_{xx} \frac{\partial^2 h}{\partial x^2} + \bar{T}_{yy} \frac{\partial^2 h}{\partial y^2} \pm W_p = S_y \frac{\partial h}{\partial t}$$

1D Steady State Solutions

$$\frac{\partial}{\partial x}\left(K_{xx} h \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy} h \frac{\partial h}{\partial y}\right) + W_R - W_P = S_t \frac{\partial h}{\partial t}$$

for steady state and one dimensional flow

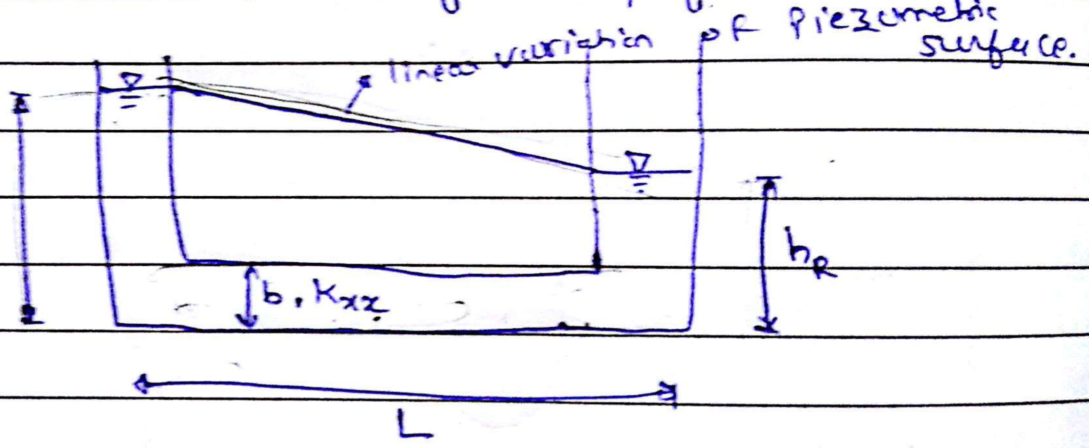
$$\frac{\partial h}{\partial t} = 0 \quad \text{and} \quad \frac{\partial h}{\partial y} = 0$$

$$\Rightarrow \quad \frac{\partial}{\partial x}\left(K_{xx} h \frac{\partial h}{\partial x}\right) + \underset{\substack{\text{Inflow} \\ \text{(source)}}}{W_R} - \underset{\substack{\text{outflow} \\ \text{(sink)}}}{W_P} = 0$$

Example:-

lets consider confined aquifer

(Two canals) at h_L and h_R



what is piezometric surface over Length 'L'

assume 1D steady state flow.

$$W_R = W_P = 0.$$

$$\frac{\partial}{\partial x}\left(K_{xx}\, b \frac{\partial h}{\partial x}\right) = 0$$

$$\Rightarrow \quad K_{xx}\, b \frac{\partial^2 h}{\partial x^2} = 0$$

so, $$\frac{\partial^2 h}{\partial x^2} = 0$$

$$h = C_1 x + C_2$$

at $x = 0$, $h = h_L$ $\Rightarrow$ $C_2 = h_L$

So, $$\boxed{h = -\frac{(h_L - h_R)}{L}x + h_L}$$

Piezometric surface will be linear

If we consider unconfined aquifer

$$\frac{\partial}{\partial x}\left(K_{xx} h \frac{\partial h}{\partial x}\right) = 0$$

$$K_{xx}\left[h\frac{\partial^2 h}{\partial x^2} + \left(\frac{\partial h}{\partial x}\right)^2\right] = 0$$

$$h\frac{\partial^2 h}{\partial x^2} + \left(\frac{\partial h}{\partial x}\right)^2 = 0$$

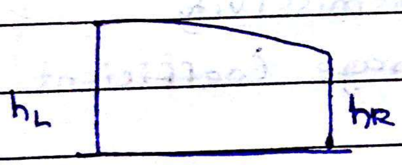
$$\frac{\partial h}{\partial x} = t \Rightarrow \frac{\partial^2 h}{\partial x^2} = \frac{\partial t}{\partial x}$$

$$\Rightarrow h\frac{\partial t}{\partial x} + t^2 = 0$$

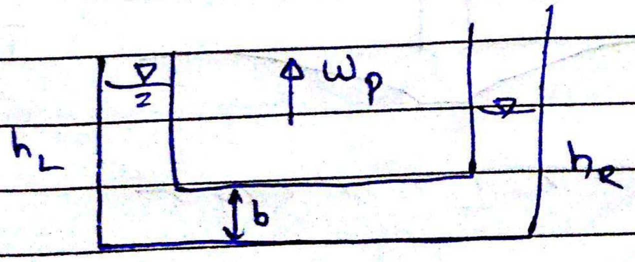
Solution is $$\boxed{h^2 = \frac{(h_R^2 - h_L^2)}{L}x + h_L^2}$$

Parabolic Variation

$$\boxed{h = \sqrt{-\frac{(h_L^2 - h_R^2)}{L}x + h_L^2}}$$



If pumping is taking place (confined aquifer)



$W_p \neq 0$ (in this case)

$$\frac{\partial}{\partial x}\left(K_{xx} b \frac{\partial h}{\partial x}\right) - W_p = 0$$

$$\Rightarrow K_{xx} b \frac{\partial^2 h}{\partial x^2} = W_p$$

Date : ___/___/___

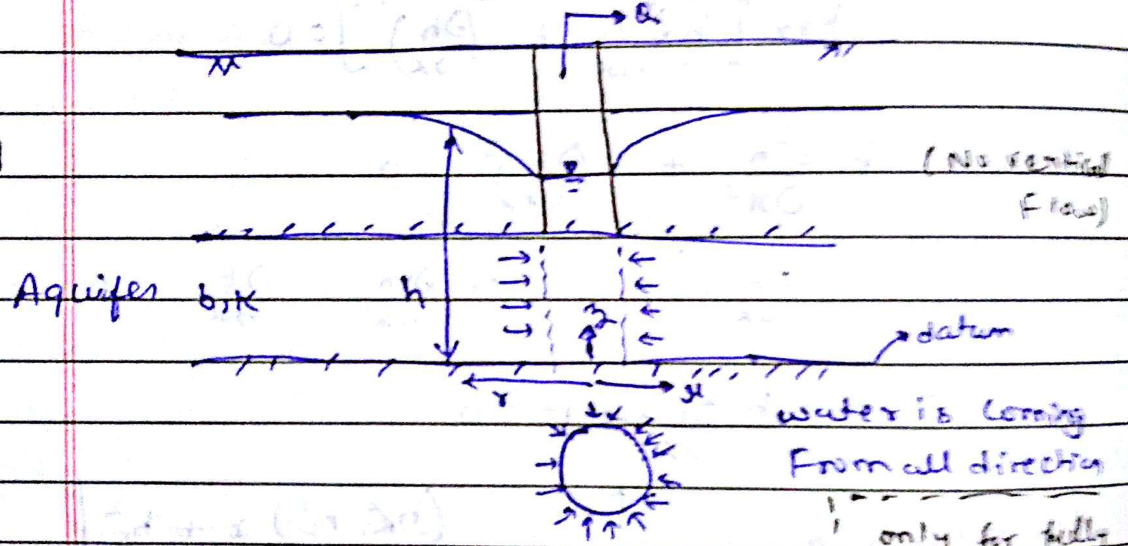
$$K_{xx} b \frac{\partial h}{\partial x} = w_p x + C_1$$

$$K_{xx} b\, h = \frac{w_p x^2}{2} + C_1 x + C_2$$

$$\boxed{h = \frac{1}{b K_{xx}} \left[\frac{w_p x^2}{2} + C_1 x + C_2 \right]}$$

Well Hydraulics:-

lets consider confined aquifer, isotropic



only for fully Penetrating well, there will not be vertical flow

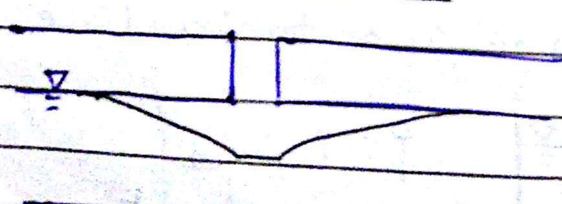
Piezometric level $= h = f(b, K, r, t)$ and also $f(S_t)$

so, $h = f(T, S_t, r, t)$

T = Transmissivity

S_t = Storage Coefficient

For case of Unconfined aquifer



There will be a vertical flow

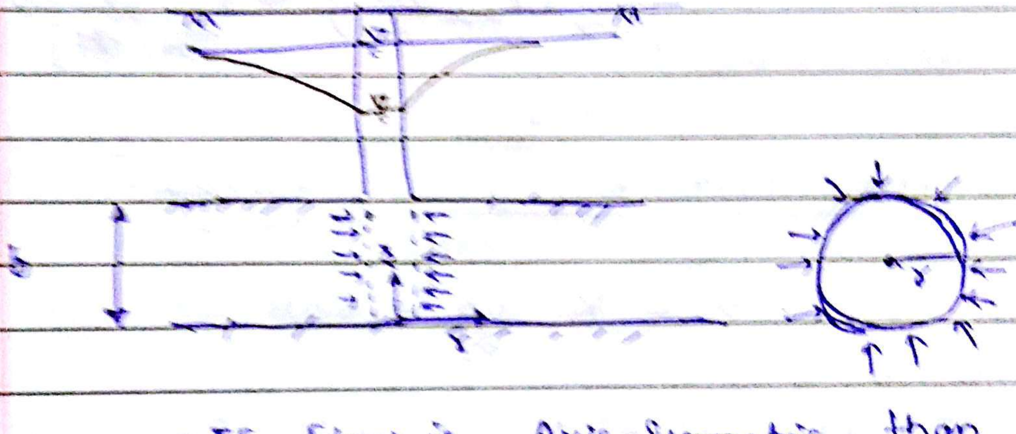
$h = f(K, S_y, r, z, t)$

UPCP

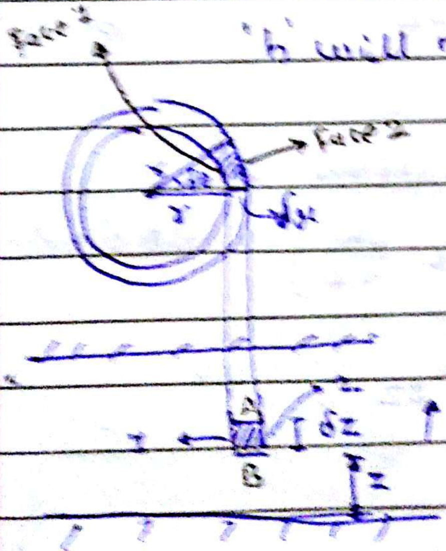
For Fully Penetrating well (For Confined)
$\Rightarrow h = f(T, S_c, r, t)$

For Partially Penetrating well (For unconfined)
$h = f(T, S_c, r, z, t)$

For Confined aquifer (No Vertical Flow)
$h = f(x, y, z, t) \rightarrow$ Cartesian Coordinates
$h = f(r, \theta, z, t) \rightarrow$ Polar Coordinates



If Flow is Axis Symmetric than 'h' will not depend on 'θ' $\boxed{h = f(r, z, t)}$



$K_{xx} \Rightarrow$ Hydraulic Conductivity

Inflow From Face 1 Per unit time
$$\Rightarrow -K_{xx}\left(\frac{\partial h}{\partial x}\right)(r\,\delta\theta \times \delta z)$$

Outflow From Face 2 Per unit time
$$\Rightarrow -\left[K_{xx}\left(\frac{\partial h}{\partial x}\right)(r\,\delta\theta \times \delta z) + \frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\,x\right)\delta\theta\,\delta z \cdot \delta x\right]$$

Inflow − outflow in r direction

$$\Rightarrow \frac{\partial}{\partial r}\left[\left(K_{rr}\frac{\partial h}{\partial r} r\,\delta\theta\,\delta z\right)\delta r\right]$$

$$\Rightarrow \frac{\partial}{\partial r}\left[K_{rr}\, r\frac{\partial h}{\partial r}\right] \delta r\,\delta\theta\,\delta z$$

$$\Rightarrow \cancel{r\frac{\partial}{\partial r}\left[K_{rr}\frac{\partial h}{}\right.}$$

$$\Rightarrow \left[\frac{\partial}{\partial r}\left(K_{rr}\frac{\partial h}{\partial r}\right) + \frac{K_{rr}}{r}\frac{\partial h}{\partial r}\right]\delta V$$

Inflow from faze 'z' in z direction:-

$$\Rightarrow -K_{zz}\frac{\partial h}{\partial z}\, r\,\delta\theta\,\delta r$$

Out flow $\Rightarrow -\left[K_{zz}\frac{\partial h}{\partial z} r\,\delta\theta\,\delta r + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z} r\,\delta\theta\,\delta r\right)\right.$

Change = Inflow − outflow

$$\Rightarrow \frac{\partial}{\partial z}\left[K_{zz}\frac{\partial h}{\partial z}\right] r\,\delta\theta\,\delta r\,\delta z$$

$$\Rightarrow \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)\delta V$$

Net (Inflow − outflow) = Sum (~~of~~ from both direction)

$$\Rightarrow \left[\frac{K_{rr}}{r}\frac{\partial h}{\partial r} + \frac{\partial}{\partial r}\left(K_{rr}\frac{\partial h}{\partial r}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)\right]\delta V$$

Date : ___/___/___

$$\frac{K_{rr}}{r}\frac{\partial h}{\partial r} + \frac{\partial}{\partial x}\left(K_{rr}\frac{\partial h}{\partial r}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) = S_s\frac{\partial h}{\partial t}$$

If we will consider the source and sink term than, sink term will be subtracted and source term will be added.

$$\boxed{\underbrace{\frac{K_{rr}}{r}\frac{\partial h}{\partial r}}_{\text{Radial Effect of flow}} + \frac{\partial}{\partial x}\left(K_{rr}\frac{\partial h}{\partial r}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) + \overset{\text{ReSource}}{W_R} - \overset{\text{Sink}}{W_S} = S_s\frac{\partial h}{\partial t}}$$

General Ground water Equation (can be used for confined and unconfined)

For confined Aquifer Equation of ground water flow if well is fully in penetrating aquifer

no vertical flow possible

$$\frac{\partial h}{\partial z} = 0$$

$W_R, W_S \Rightarrow$ Per unit Volume

$W_R', W_S' \Rightarrow$ Per unit area

$$\frac{b\,K_{rr}}{r}\frac{\partial h}{\partial r} + b\frac{\partial}{\partial x}\left(K_{rr}\frac{\partial h}{\partial r}\right) + (W_R' - W_S')b = S_s b\frac{\partial h}{\partial t}$$

$$\Rightarrow \frac{T_{rr}}{r}\frac{\partial h}{\partial r} + T_{rr}\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial r}\right) + (W_R' - W_S')b = S_s b\frac{\partial h}{\partial t}$$

$$\Rightarrow \boxed{\frac{T_{rr}}{r}\frac{\partial h}{\partial r} + T_{rr}\frac{\partial^2 h}{\partial r^2} + (W_R' - W_S')b = S_t\frac{\partial h}{\partial t}}$$

Date : ___/___/___

For unconfined aquifer

$$\frac{T_{rr}}{r}\frac{\partial h}{\partial r} + \frac{\partial}{\partial x}\left(T_{rr}\frac{\partial h}{\partial r}\right) + \frac{\partial}{\partial z}\left(K_{zz}\, b\frac{\partial h}{\partial z}\right) + (W_R^* - W_S^*) = (S_y + S_t)\frac{\partial h}{\partial t}$$

This is varying

~~K_{rr}~~

So,

$$\frac{h K_{rr}}{r}\frac{\partial h}{\partial r} + \frac{\partial}{\partial x}\left(K_{rr} h \frac{\partial h}{\partial r}\right) + \frac{\partial}{\partial z}\left(K_{zz} h \frac{\partial h}{\partial z}\right) + W_R' - W_S' = S_y\frac{\partial h}{\partial t}$$

for unconfined $S_t \simeq 0$

If change in water Table is very low than vertical term can be neglected

So, $\frac{\partial h}{\partial z} = 0$

$$\boxed{\Rightarrow h\frac{K_{rr}}{r}\frac{\partial h}{\partial r} + \frac{\partial}{\partial x}\left(K_{rr} h\frac{\partial h}{\partial r}\right) + (W_R' - W_S') = S_y\frac{\partial h}{\partial t}}$$

Non linear Equation ⇐

If flow is steady than $\frac{\partial h}{\partial t} = 0$

For Axis Symmetric flow, unconfined aquifer, unsteady state

* when it will reach steady state

* अगर चारों में River होता। (River water level will not change much)

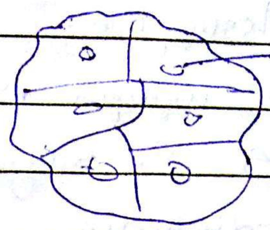
* Stream Aquifer Interaction

5/2/17

Ground water Problems can be divided in two Scales

(1) Regional scale →

(2) Local Scale



How much Pumping we should do ?

(Regional Scale Problems)

we can't assume aquifer Parameters same for whole Regional area. so we divide the area into smaller local area and than we calculate aquifer Parameters for local area.

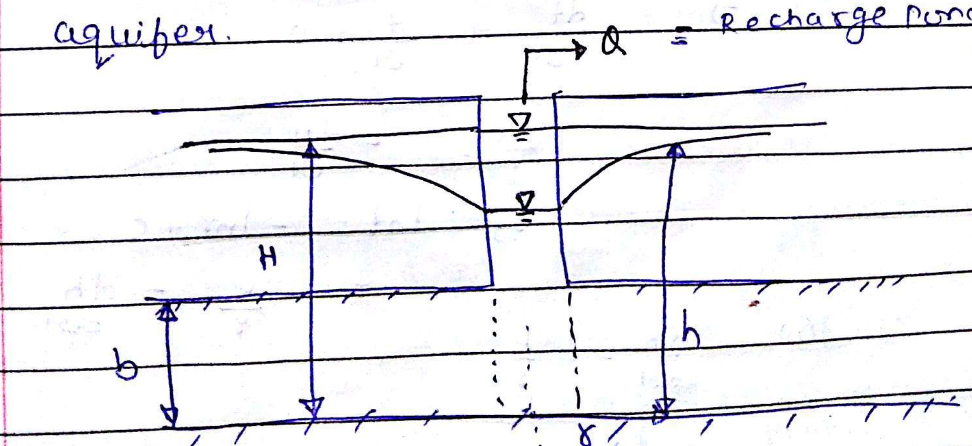
~~so~~, analysis of the local area is called local scal ground water Problems.

* So, All well field Problems comes out in the range of local scale Problems
* To solve Regional scale Problems you need to understand the local scal Problems.

Well Hydraulics (Steady State)

lets consider a well in a confined aquifer.

$\to Q$ = Recharge Pond के द्वारा जितना दिया जा रहा है। (Steady state)



Assumption (UPCP) * If I have Recharge Pond at a distance 'R' than system will Reach steady state.

$h(r) = ?$

Assumption:-

1) aquifer is Homogenous and isotropic
2) Recharge Boundary Exist at a Radial distance 'R' from the well.
3) Well is fully Penetrating the aquifer. (flow will be Horizontal only)
4) Discharge, Q is constant throughout the Pumping. (practically not Possible)

Governing equation

$$\frac{T_{rr}}{r}\quad \frac{\partial}{\partial r}\left(k_{rr} b \frac{\partial h}{\partial r}\right) + \frac{k_{rr} b}{r}\frac{\partial h}{\partial r} - \cancel{R}^{0} = S_t \cancel{\frac{\partial h}{\partial t}}^{0}$$

$$\Rightarrow T_{rr}\frac{\partial^2 h}{\partial r^2} + \frac{T_{rr}}{r}\frac{\partial h}{\partial r} = 0$$

$$T_{rr}\left[\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r}\right] = 0$$

general Solution of this equation

$$\frac{d^2h}{dr^2} + \frac{1}{r}\frac{dh}{dr} = 0$$

Put $\frac{dh}{dr} = t$, $\frac{d^2h}{dr^2} = \frac{dt}{dr}$

$$\Rightarrow \frac{dt}{dr} + \frac{t}{r} = 0$$

$$\frac{dt}{t} = -\frac{dr}{r}$$

$$\Rightarrow \ln t = -\ln r + c$$

$$t = \frac{k}{r} = \frac{dh}{dr}$$

So, $\frac{dh}{dr} = \frac{k}{r}$

$$\Rightarrow \boxed{h = k\ln r + C}$$

$$\boxed{h = A\ln r + B} \qquad \boxed{H = A\ln R + B} \longrightarrow \text{at } r = R,\ h = H$$

$$Q = KiA$$

$$Q = K\frac{dh}{dr}(2\pi r b)$$

$$Q = 2\pi T\, r\frac{dh}{dr}$$

$$r\frac{dh}{dr} = \frac{Q}{2\pi T}$$

$$\Rightarrow \frac{dh}{dr} = \frac{A}{r}$$

So, $$r\left(\frac{A}{r}\right) = \frac{Q}{2\pi T}$$

$$\boxed{A = \frac{Q}{2\pi T}}$$

So, $$H = \frac{Q}{2\pi T}\ln R + B$$

$$\boxed{B = H - \frac{Q}{2\pi T}\ln R}$$

So, $$h = \frac{Q}{2\pi T}\ln r + H - \frac{Q}{2\pi T}\ln R$$

$$h = \frac{Q}{2\pi T}\ln r/R + H$$

$$\Rightarrow h - H = \frac{Q}{2\pi T}\ln r/R$$

So, $$\boxed{Q = \frac{2\pi T(h-H)}{\ln r/R}}$$

$$\boxed{Q = \frac{2\pi T(H-h)}{\ln R/r}}$$

$H - h =$ drawdown $= s$ at a distance r

$\boxed{R = \text{Radius of Influence}}$ => beyond which effect of Pumping is faded up.

Date : / /

→ fully Penetrating well

so, $$\boxed{S = \frac{Q}{2\pi T} \ln(R/r)}$$

Confined aquifer

Steady state

Isotropic and Homogenous

Drawdown

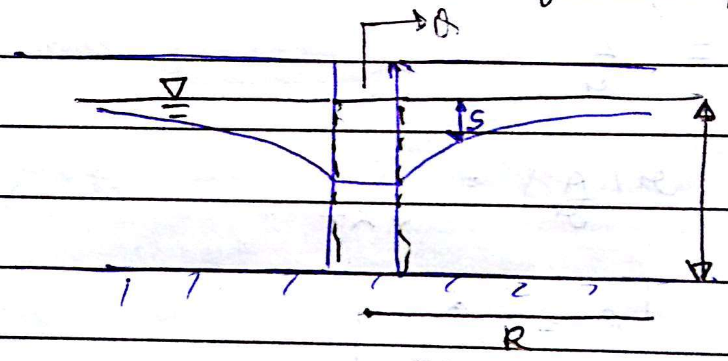
(Theims equation for confined aquifer)

$S \propto Q$ (linear Relationship)

(and system is linear)

Now lets consider Unconfined aquifer (Steady State)

for Unconfined



assumptions are same as in Previous case

Governing equation

$$\frac{K_{rr}}{r} h \frac{dh}{dr} + K_{rr} \frac{d}{dr}\left(h \frac{dh}{dr}\right) = 0$$

$$\Rightarrow K_{rr}\left[\frac{h}{r}\frac{dh}{dr} + h\frac{d^2h}{dr^2} + \left(\frac{dh}{dr}\right)^2\right] = 0$$

$$\frac{h}{r}\frac{dh}{dr} + h\frac{d^2h}{dr^2} + \left(\frac{dh}{dr}\right)^2 = 0$$

How you will solve above equation.

let assume $h^2 = \phi$

$$2h\frac{dh}{dr} = \frac{d\phi}{dr}$$

31

$$\Rightarrow \quad h^2 = A\ln r + B$$

$$H^2 = A\ln R + B \quad \Rightarrow \text{at } r = R,\ h = H$$

Final equation will be

$$H^2 - h^2 = \frac{Q}{\pi K}\ln(R/r) \quad \Rightarrow \boxed{Q = \frac{(H^2-h^2)\pi K}{\ln(R/r)}}$$

$$\Rightarrow (H+h)(H-h) = \frac{Q}{\pi K}\ln(R/r)$$

$$\Rightarrow (S+2h)(S) = \frac{Q}{\pi K}\ln(R/r)$$

$$\Rightarrow \boxed{S^2 + 2hS = \frac{Q}{\pi K}\ln(R/r)} \rightarrow \text{Non linear Relationship}$$

If $S << H$

$$\Rightarrow (2H-S)(S) = \frac{Q}{\pi K}\ln(R/r)$$

$$\Rightarrow S << H \text{ than } (2H-S) \simeq 2H$$

$$\Rightarrow 2HS = \frac{Q}{\pi K}\ln(R/r)$$

for $\boxed{S << H}$ $\boxed{S = \frac{Q}{2\pi K H}\ln(R/r)}$ linear Relationship

$KH = T$ {equivalent Transmissivity} for $S << H$

3/2/2017

* If water Table elevation change is very less compare to saturated thickness than unconfined aquifer can be treate as confined aquifer for steady state

$$S = \frac{Q}{2\pi T}\ln(R/r)$$

$\hookrightarrow KH = T$ (for unconfined)

UPCP

$S(r_1) = S_1$
$S(r_2) = S_2$] → Two observation well

$$S_1 = \frac{Q}{2\pi T} \ln(R/r_1)$$

$$S_2 = \frac{Q}{2\pi T} \ln(R/r_2)$$

$$S_1 - S_2 = \frac{Q}{2\pi T} \left(\ln\left(\frac{r_2}{r_1}\right)\right)$$

$$\boxed{Q = \frac{2\pi T (S_1 - S_2)}{\ln(r_2/r_1)}} \qquad \boxed{T = \frac{Q \ln(r_2/r_1)}{2\pi (S_1 - S_2)}}$$

* you cannot get any idea of Storage Parameter (S_t, S_y) from steady state analysis because storage at steady state is zero. no change in storage at steady state.

Drawbacks of Steady state analysis
- No storage Parameter
- It takes times to achieve steady state in practical
- not used in commonly.

* Unsteady State Solutions:-

For confined aquifer
assumption:- → aquifer is isotropic and Homogeneous.
→ Infinite in areal extent
→ well fully penetrates the aquifer
→ discharge is constant.
→ No Resource and sink

Date : __/__/__

Governing equation:-

$$\frac{T_{rr}}{r}\frac{\partial h}{\partial r} + T_{rr}\frac{\partial^2 h}{\partial r^2} = S_t \frac{\partial h}{\partial t}$$

$$\Rightarrow \frac{T}{r}\frac{\partial h}{\partial r} + T\frac{\partial^2 h}{\partial r^2} = S_t \frac{\partial h}{\partial t}$$

we need 2 condition in r (2 degree in r)

1 condition in t (1^{st} degree in t)

initially piezometric level $= H$

at $t = 0$, $h = H$ let r_w is the Radius of well.

$r \neq 0, ..$

Convert eqn in 's'

$$S = H - h$$

$$\Rightarrow -\frac{T}{r}\frac{\partial S}{\partial r} - T\frac{\partial^2 S}{\partial r^2} = -S_t \frac{\partial S}{\partial t}$$

$$\Rightarrow \frac{T}{r}\frac{\partial S}{\partial r} + T\frac{\partial^2 S}{\partial r^2} = S_t \frac{\partial S}{\partial t}$$

Initial Condition

$t = 0$, $S = 0$ $\forall\, r \in [r_w, \infty)$

Boundary Condition

$t > 0$ $r \to \infty$, $h \to H$, $S = 0$

$t > 0$ $r = r_w$, $Q = -K\left(-\frac{\partial h}{\partial r}\right)(2\pi r_w b)$

$$Q = 2\pi T r_w \left(\frac{\partial h}{\partial r}\right)$$

$$\boxed{\frac{Q}{2\pi T} = -r_w \frac{\partial S}{\partial r}}, \text{ at } r = r_w$$

Theis solution:- (for confined aquifer, unsteady state)

$$S = \frac{Q}{4\pi T} * W(u)$$

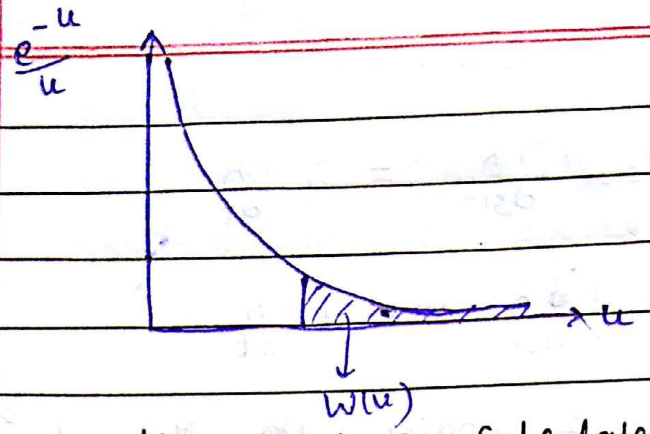
$W(u)$ = well function for confined aquifer

$$W(u) = \int_u^\infty \frac{e^{-u}}{u}\, du$$

and where $u = \frac{r^2 S_t}{4Tt}$ = non dimensional

UPCP

Date : ___/___/___



usually W(u) is calculated from tables. (standard Table)

Series Solution.

$$\boxed{W(u) = -0.5772 - \ln(u) + u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \dots}$$

If $u < 0.01$

Cooper-Jacob $\boxed{W(u) \simeq -0.5772 - \ln(u)}$

$$W(u) = \ln(0.5614) - \ln(u)$$

$$W(u) = \ln\left(\frac{0.5614}{u}\right)$$

Cooper-Jacob expression for $\boxed{u < 0.01}$ $\boxed{W(u) = 2.303 \log\left(\frac{2.2456\, Tt}{r^2 S_t}\right)}$

So, $\boxed{S = \frac{2.303 Q}{4\pi T} \log\left(\frac{2.2456\, Tt}{r^2 S_t}\right)}$

System is linear

Linear Relationship b/w S and Q

Linear systems:-

→ System is said to be linear if change in Input is equal to change in output

$I_1 \rightarrow O_1$

$I_2 \rightarrow O_2$

$I_1 + I_2 \rightarrow O_1 + O_2$

→ $mI_1 \rightarrow mO_1$

* Principal of Superposition can be used.

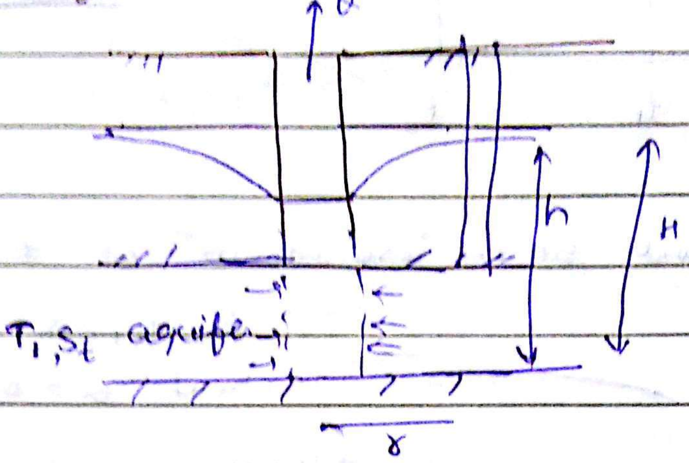
Then system will be linear $\boxed{I = KQ}$

Input, constant, output

14/3/17

Date : ___/___/___

Unsteady Radial Flow towards a well in confined aquifer :-



(1) Isotropic Homogenous

(2) well fully Penetrating the aquifer

(3) Q is constant

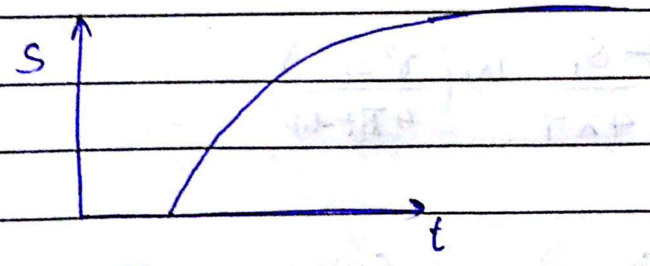
$h = f(r, t)$?

$$T\left(\frac{1}{r}\frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2}\right) = S_t \frac{\partial h}{\partial t}$$

$$S = H - h$$

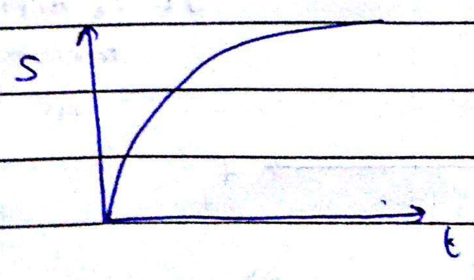
$$T\left(\frac{1}{r}\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}\right) = S_t \frac{\partial s}{\partial t}$$

(already discussed in previous class)

* 

(Typical Pattern of 'S' w.r.t 't'.

* Maximum drawdown at well phase $\boxed{r = r_w}$



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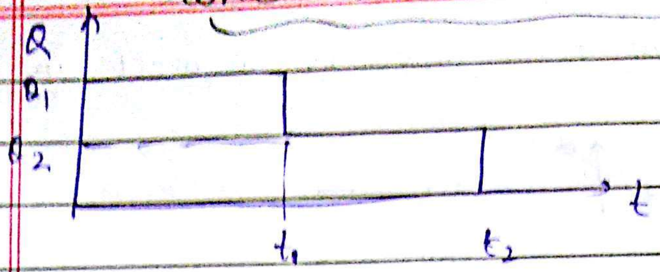
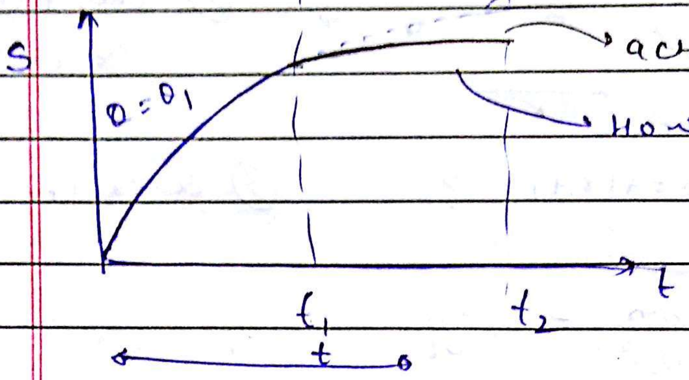
Important

When 'Q' is not constant all the time



So, what will be the drawdown at $t = \frac{t_1+t_2}{2}$?



when $Q_2 = Q_1 = Q$

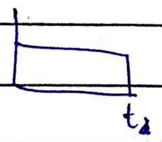
actual $Q_1 > Q_2 < Q_1$

How you will get this curve

(at $r = r_w$)

$$S = \left[t \in (t_1, t_2)\right] = \cancel{\frac{Q \times W(u)}{4\pi T}}$$

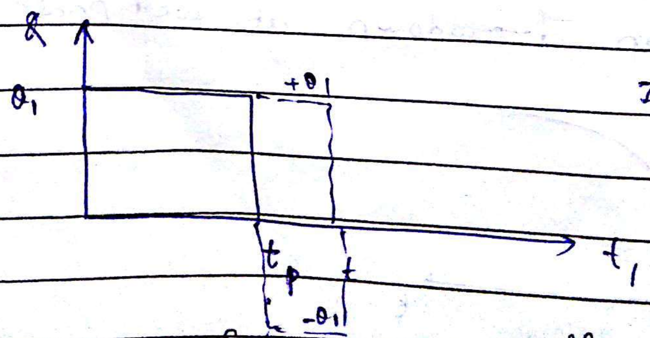
$$= \frac{Q_1}{4\pi T} W\left(\frac{r^2 S_t}{4\pi t}\right) + \frac{Q_2}{4\pi T} W\left(\frac{r^2 S_t}{4T(t-t_1)}\right)$$



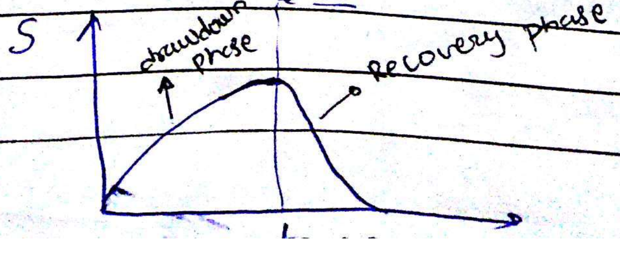
$$-\frac{Q_1}{4\pi T} W\left(\frac{r^2 S_t}{4T(t-t_1)}\right)$$

Recovery Test :-

(when we only have pumping well)



It is difficult to measure drawdown at well phase so we use Recovery Test



at $r = r_w$

Practically recovery time is less

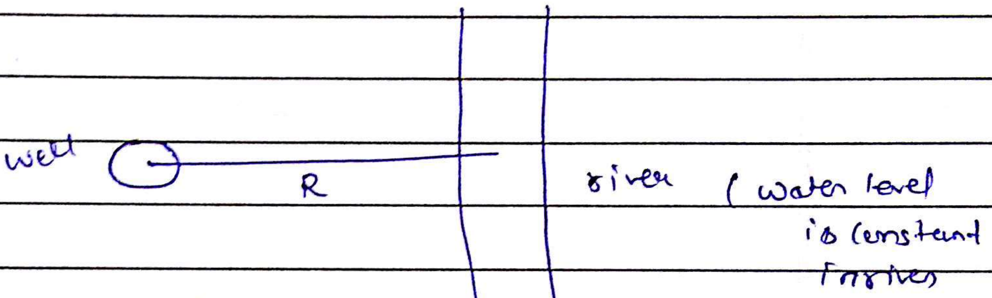
theoritically = infinite

What is $s(t > t_p)$

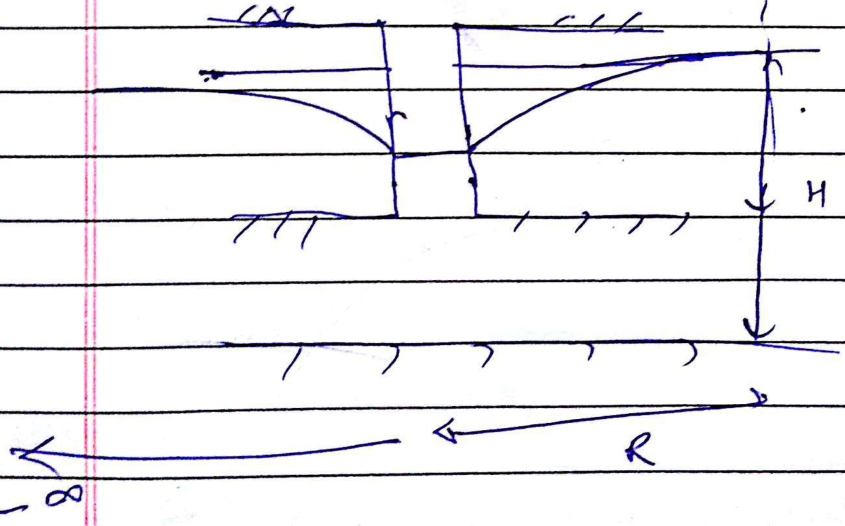
$$t > t_p \quad s(t) = \frac{Q_1}{4\pi T} W\left(\frac{r^2 S_t}{4Tt}\right) - \frac{Q_1}{4\pi T} W\left[\frac{r^2 S_t}{4T(t-t_p)}\right]$$

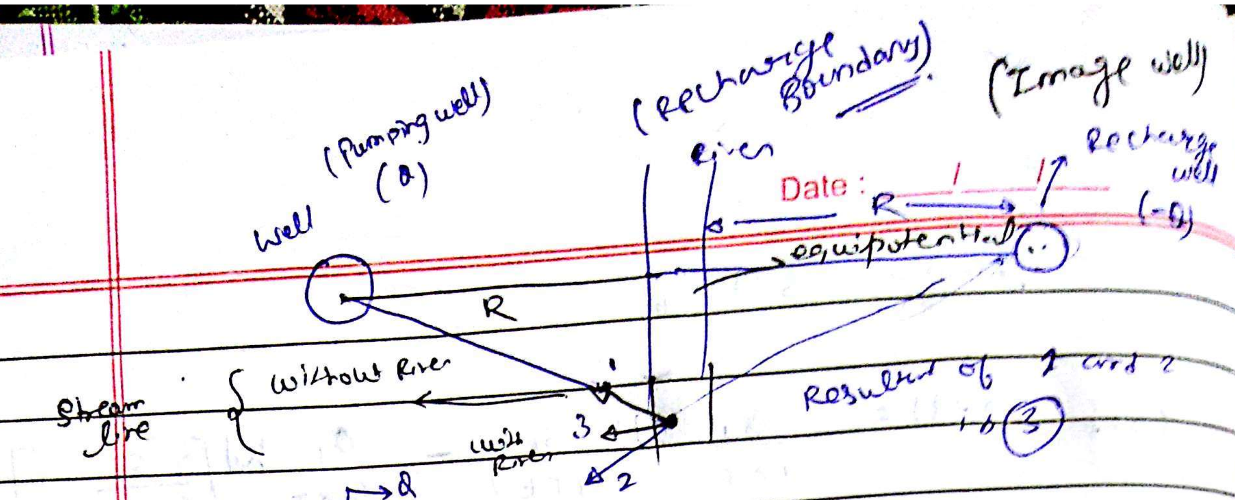
$$t < t_p \quad s(t) = \frac{Q_1}{4\pi T} W\left(\frac{r^2 S_t}{4Tt}\right)$$

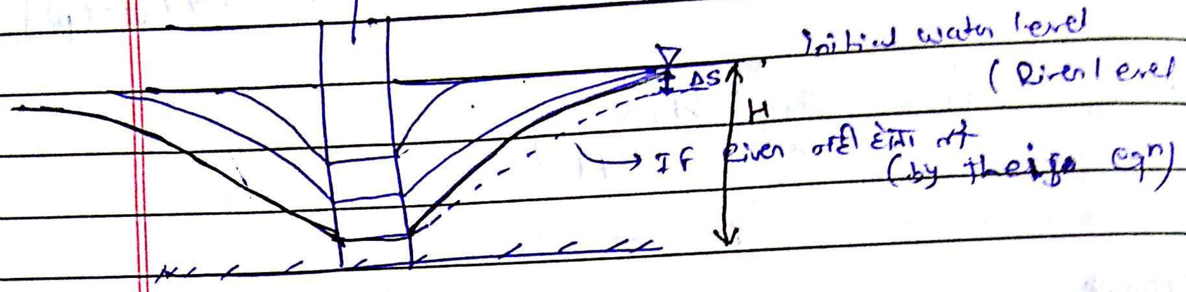
Example



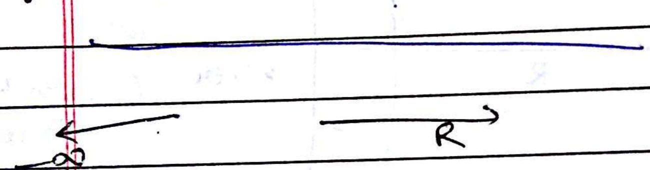
at r = R, I have a river







Aquifer, T, S_t



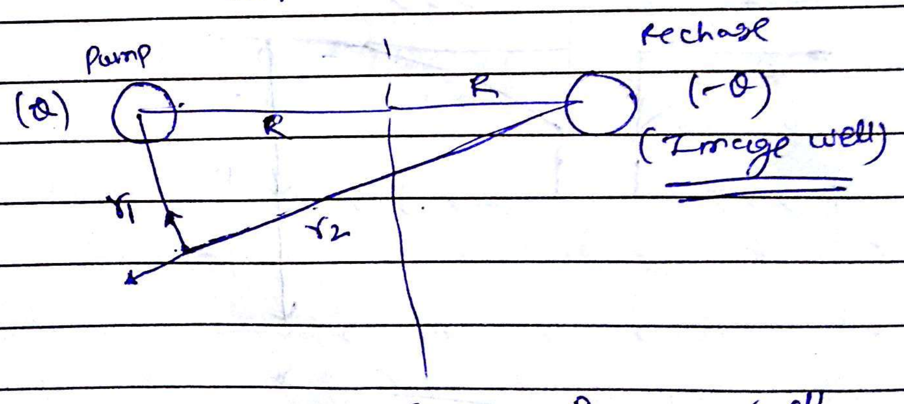
Pumping well + River

= equivalent to

Pump + Recharge

System

$$S = \frac{Q}{4\pi T} W(u)$$

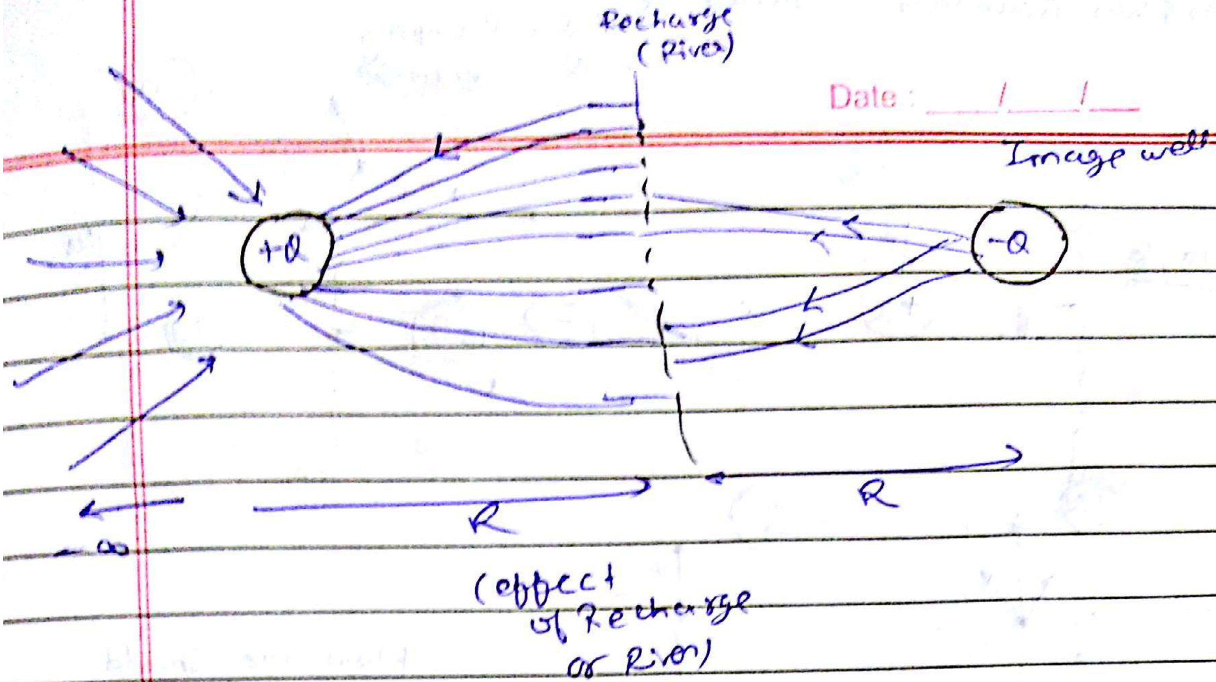


drawdown at r_1 from Pumping well

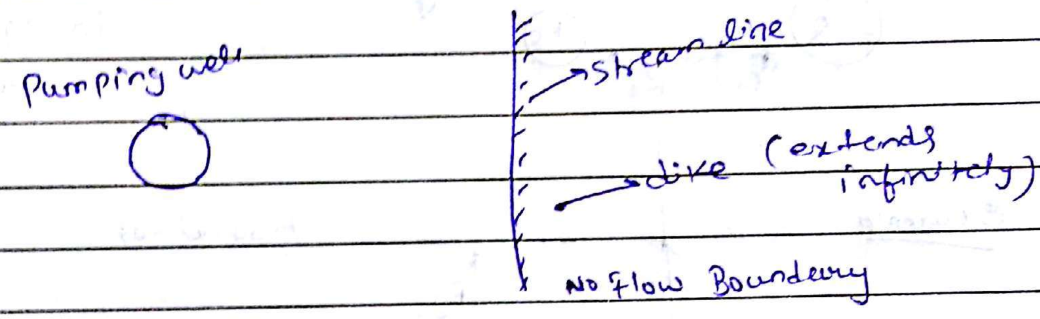
Taking care of increase of water level due to River or Recharge ~~level~~.

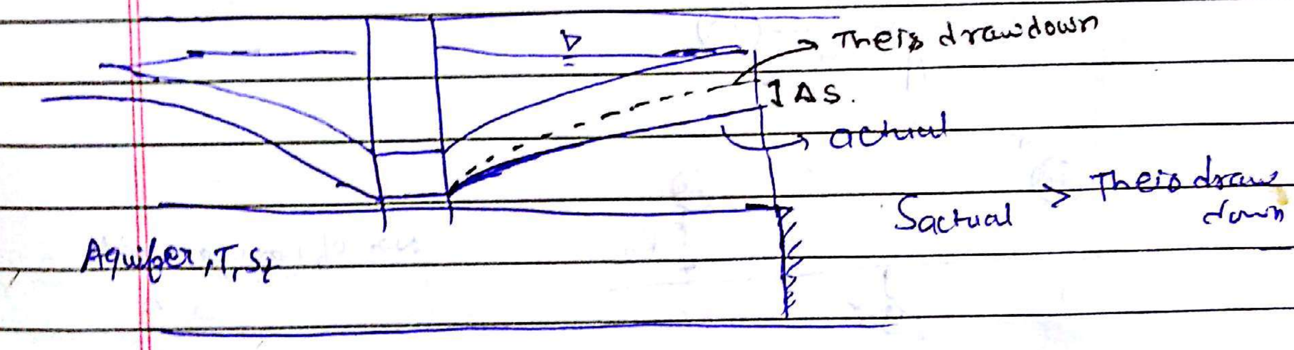
$$S = \frac{Q}{4\pi T} W\left(\frac{r_1^2 S_t}{4Tt}\right) - \frac{Q}{4\pi T} W\left(\frac{r_2^2 S_t}{4Tt}\right)$$

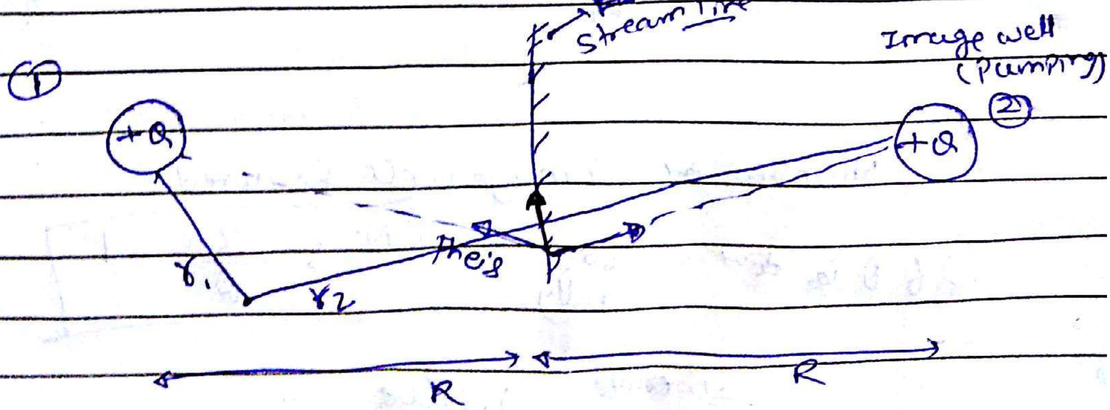
Flow line Pattern



Example







drawdown at r_1 distance from well ①

$$S = \frac{Q}{4\pi T} W\left(\frac{r_1^2 S_t}{4Tt}\right) + \frac{Q}{4\pi T} W\left(\frac{r_2^2 S_t}{4Tt}\right)$$

Recharge Boundary : Equipotential

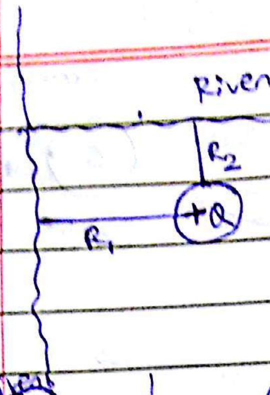
No Flow Boundary ⇒ Stream line

+Q = Pumping

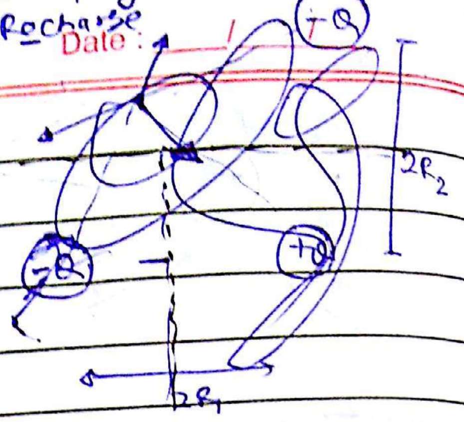
−Q = Recharge

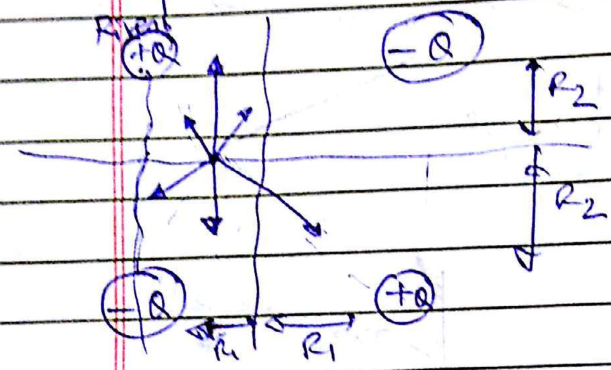
Date :

example



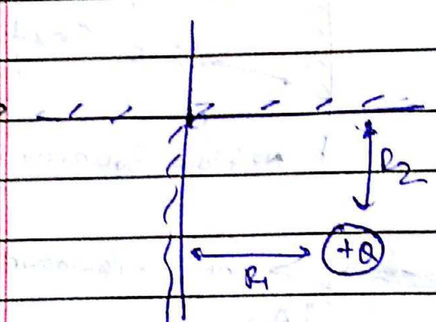
⇒





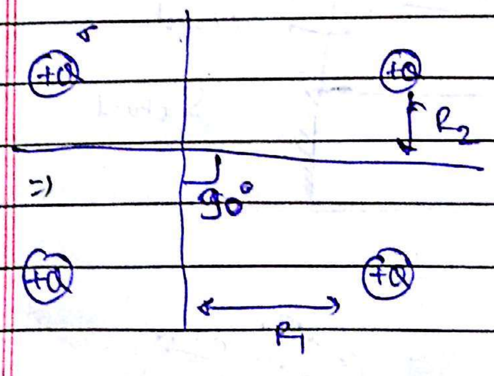
Flow line should be perpendicular to equipotential line.

example



two dikes

⇒



No. of Image well ⇒ 3

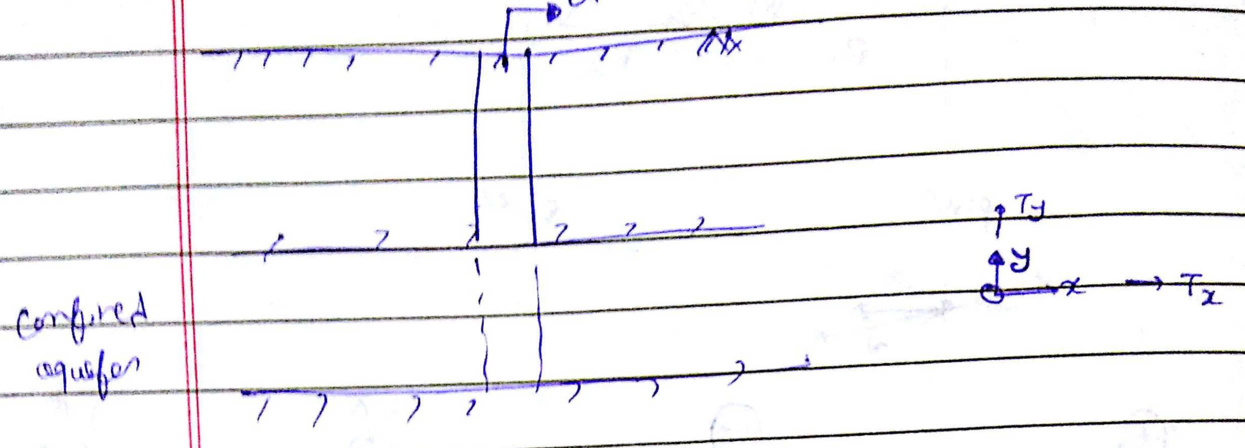
Number of Image well Required

If θ is divides 360 fully

$$\boxed{N_i = \frac{360}{\theta} - 1}$$

otherwise Infinite.

Anisotropic Well :-



Governing equation

Radial anisotropy $T_r = T_x = T_y$

$$T_r\left(\frac{1}{r}\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}\right) = S_t \frac{\partial s}{\partial t}$$ [for Isotropic aquifer]

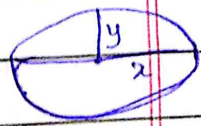
In Cartesian coordinate

$$T_x \frac{\partial^2 s}{\partial x^2} + T_y \frac{\partial^2 s}{\partial y^2} = S_t \frac{\partial s}{\partial t}$$ (anisotropic) $T_x \neq T_y$

if $T_x = T_y = T$

$$\Rightarrow T\left[\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2}\right] = S_t \frac{\partial s}{\partial t}$$ (isotropic)

assume, x, y principal anisotropy direction coincide with x, y direction



So, governing eq^n

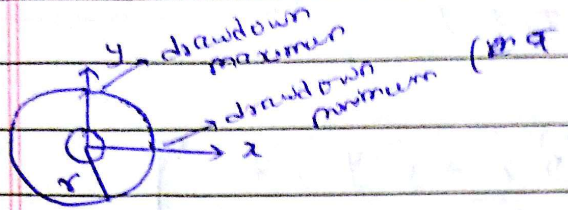
$$T_x \frac{\partial^2 s}{\partial x^2} + T_y \frac{\partial^2 s}{\partial y^2} = S_t \frac{\partial h}{\partial t}$$

If Principal axis does not coincide with 'x, y direction.

Now, assume, Principle axes of anisotropy is coinciding with principal directions.

Date : ___/___/___

$$T_x \frac{\partial^2 s}{\partial x^2} + T_y \frac{\partial^2 s}{\partial y^2} = S_t \frac{\partial h}{\partial t} \qquad \boxed{T_x > T_y}$$



we need to convert in equivalent isotropic domain.

let $X = \left(\frac{T_y}{T_x}\right)^{1/4} x$

$Y = \left(\frac{T_x}{T_y}\right)^{1/4} y$

$\Rightarrow \cancel{T_x \left(\frac{T_y}{T_x}\right)^{1/4} \frac{\partial^2 s}{\partial X^2} + T_y \left(\frac{T_x}{T_y}\right)^{1/4} \frac{\partial^2 s}{\partial Y^2} = S_t \frac{\partial h}{\partial t}}$

??

$$\sqrt{T_x T_y}\left[\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2}\right] = S_t \frac{\partial h}{\partial t}$$

$$T_r\left[\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2}\right] = S_t \frac{\partial h}{\partial t}$$

$r^2 = X^2 + Y^2$

$r^2 = x^2 \sqrt{\frac{T_y}{T_x}} + y^2 \sqrt{\frac{T_x}{T_y}}$

$$\Rightarrow T_r\left[\frac{1}{r}\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}\right] = S_t \frac{\partial s}{\partial t}$$

$$\Rightarrow \boxed{\sqrt{T_x T_y}\left[\frac{1}{r}\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}\right] = S_t \frac{\partial s}{\partial t}}$$

where $r =$

degree of anisotropy $= \frac{T_y}{T_x} = D$

$\boxed{r^2 = D^{\frac{1}{2}} x^2 + D^{-1/2} y^2}$ $\boxed{r^2 = X^2 + Y^2}$

UPCP

Date : ___/___/___

$T_r = \sqrt{T_x T_y}$

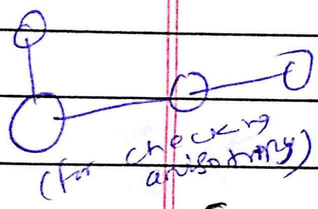
$$s = \frac{Q}{4\pi T_r} W\left(\frac{r^2 S_t}{4 T_r t}\right)$$

$$\boxed{r^2 = D^{1/2} x^2 + D^{-1/2} y^2}$$

$$s = \frac{Q}{4\pi T_r} W\left[\frac{\left(D^{1/2}x^2 + D^{-1/2}y^2\right) S_t}{4\sqrt{T_x T_y}\, t}\right]$$

Pumping Test for confined aquifer = 3 days Time (costly, expensive)

unconfined aquifer — minimum 7 days तक pumping करते। for good result

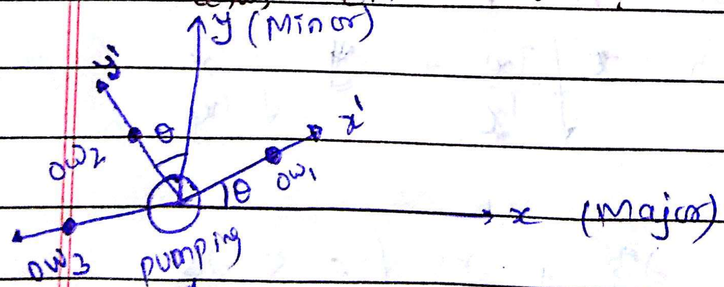


For Isotropic aquifer, Parameters = T, S_t

For anisotropic $\Rightarrow$ T, D, S_t

or T_x, T_y, S_t

* if Principle axis is not coinciding with coordinate axis then How?



OW = observation well

3 wells Required If you dont know the Major and minor direction

(इससे आगे जानने की जरूरत नहीं है।)

Now aquifer parameters T_x, T_y, S_t, θ

~~This~~ discussion ~~is key~~

Date : ___/___/___

$S = \frac{Q}{4\pi T} W(u)$ = Thies eqn)

~~both~~ we have discussed these

(1) Presence of Boundary — Image well theory

(2) Time varying discharge — Method of Superposition

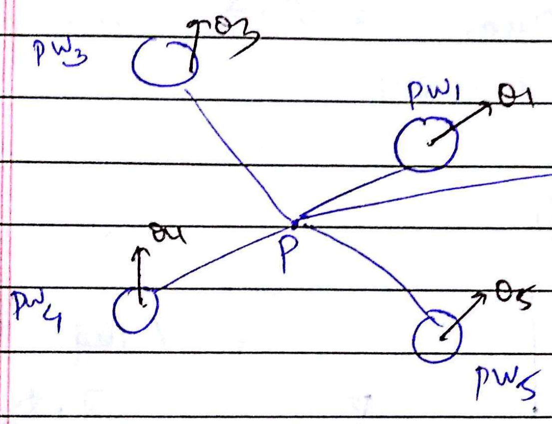
(3) Recovery

(4) Anisotropy

Now,

(5) Interference :-

pw = pumping well

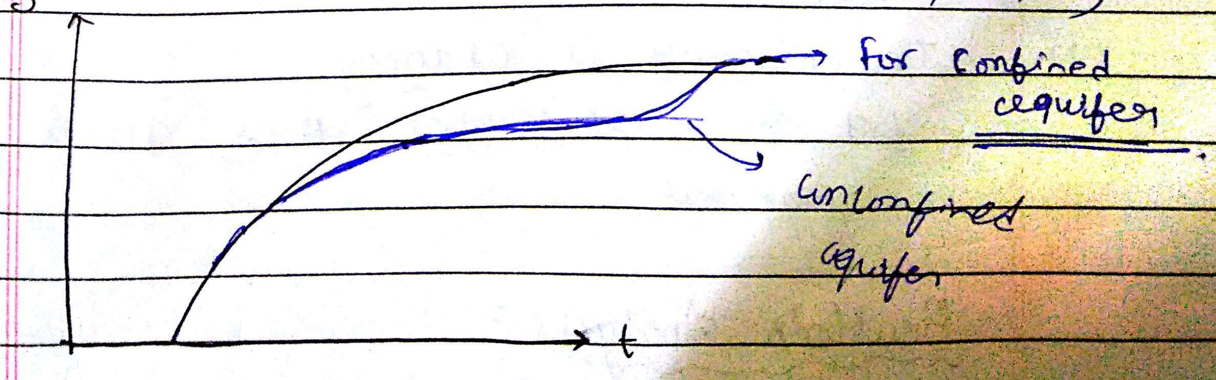


what is S_p = ? drawdown at 'p'.

$$S_p = \sum_{i=1}^{5} \frac{Q_i}{4\pi T} W\left(\frac{r_i^2 S_t}{4Tt}\right)$$

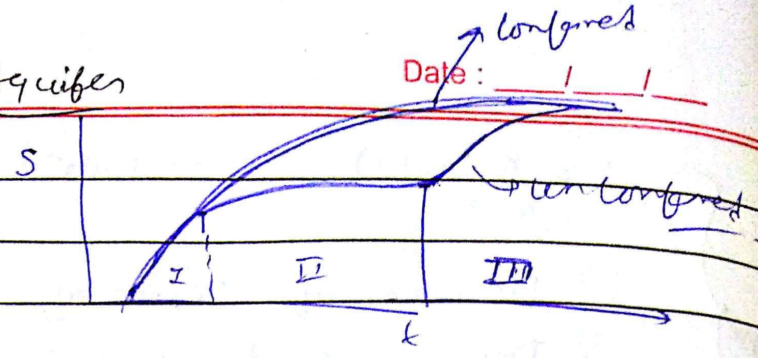
$$S_p = \sum_{i=1}^{5} \frac{Q_i}{4\pi T} W(u_i)$$

$$\boxed{S_p = \sum_{i=1}^{n} \frac{Q_i}{4\pi T} W(u_i)}$$ ⇒ confined aquifer ~~Ec~~ (simultaneous pumping)



Unconfined Aquifer

Date: ___/___/___



Parameters :- S_t, S_y, T, α

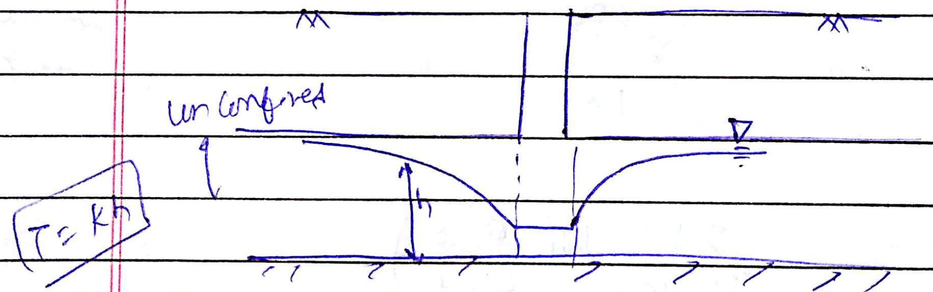
I = Instantaneous yield (S_t)

II = delayed yield (α)

III = (S_y)

long time तक pumping करनी पड़ेगी / actual and good result के लिए।

1/3/17 Unconfined Aquifer



Aquifer Parameter

T, S_t, S_y, α

(1) Domain is changing due to falling water Table

(2) Release is not instantaneous (delayed Gravity yield)

So, Thies eqn will not applicable in this case.

(3) Transmissivity is changing but for simplicity let's assume ~~Transmiss~~

Boulton's Analysis :-

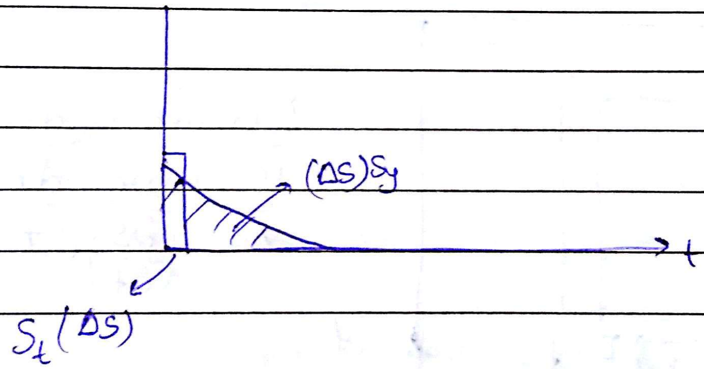
let Δs be the drawdown which has taken place.

Fall in water table = ΔS. (drop in water table)

Instantaneous yield = $(\Delta S)(S_t)$ (Per unit area)

$= S_t \Delta S$

Delayed yield (Per unit area) = $S_y \Delta S$



* If delay is negligible (Gravel Or sand is there)

~~Instaneous Total yield = $(S_t + S_y)$~~

Instantaneous yield = $(S_t + S_y)\Delta S$

Now, $u = \dfrac{r^2 (S_t + S_y)}{4Tt}$

if $S_y >> S_t$ than $u = \dfrac{r^2 S_y}{4Tt}$

* If delay is very High (Silt or Clay formation)

Instantaneous yield = $S_t \Delta S$

$u = \dfrac{r^2 S_t}{4Tt}$ (works as confined aquifer)

Isan

$$\frac{T}{r}\frac{\partial h}{\partial r} + T\frac{\partial^2 h}{\partial r^2} = S_t\frac{\partial h}{\partial t} \quad [\text{~~for confined~~}]$$

$$\frac{T}{r}\frac{\partial h}{\partial r} + \frac{\partial}{\partial r}\left(T\frac{\partial h}{\partial r}\right) = S_t\frac{\partial h}{\partial t} \quad [\text{~~Instantaneous~~}$$

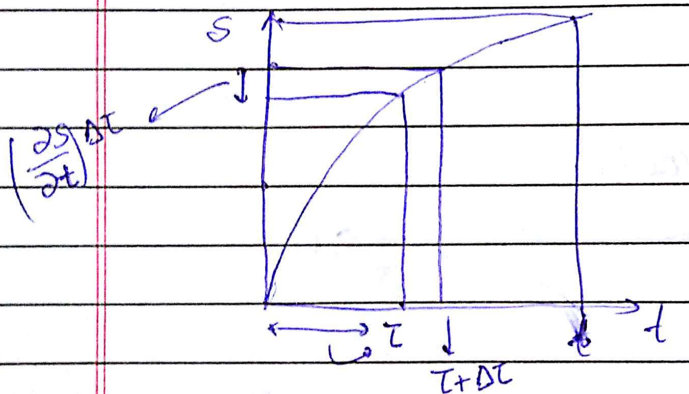
[Instantaneous release unconfined]

for unloading delayed yield $\Rightarrow$ $$\frac{T}{r}\frac{\partial h}{\partial r} + \frac{\partial}{\partial r}\left(T\frac{\partial h}{\partial r}\right) + \gamma = S_t\frac{\partial h}{\partial t}$$

γ → Contribution due to delayed drainage

$Y(t)$ $\boxed{\gamma = f(t)}$

$$Y = f(S_y, \alpha)$$



increase in drawdown in time $d\tau$

$$\Delta s = \left(\frac{\partial s}{\partial t}\right) d\tau$$

So, delayed yield = ⟸ (at time τ पर Δs increment के कारण at time t पर effect)

at time t due to this incremental drawdown ⟸ Just like Boulter's eqn $= (\alpha S_y dt)(\Delta s)$

$$= \alpha\left(S_y\frac{\partial s}{\partial t}d\tau\right)e^{-\alpha(t-\tau)}$$

$$y(t) = \int_0^t \alpha\left(S_y\frac{\partial s}{\partial t}d\tau\right)e^{-\alpha(t-\tau)}$$

??

$$y(t) = \int_0^{\tau} \alpha S_y \frac{\partial s}{\partial t} d\tau\, e^{-\alpha(t-\tau)}$$

$= 0$

Should be zero

= ??

Date : ___/___/___

Now, $\frac{T}{r}\frac{\partial h}{\partial r} + \frac{\partial}{\partial r}\left(T\frac{\partial h}{\partial r}\right) + \alpha S_y \int_0^t \frac{\partial s}{\partial t} d\tau\, e^{-\alpha(t-\tau)} = S_t \frac{\partial h}{\partial t}$

if change in water table is less compared to saturated thickness than T can be assumed to constant

$$\frac{T}{r}\frac{\partial h}{\partial r} + T\frac{\partial^2 h}{\partial r^2} + \alpha S_y \int_0^t \frac{\partial s}{\partial t} d\tau\, e^{-\alpha(t-\tau)} = S_t \frac{\partial h}{\partial t}$$

$H - h = S$

How?? ⇒

$$\frac{T}{r}\frac{\partial s}{\partial r} + T\frac{\partial^2 s}{\partial r^2} - \alpha S_y \int_0^t \frac{\partial s}{\partial t} d\tau\, e^{-\alpha(t-\tau)} = S_t \frac{\partial h}{\partial t}$$

$$\frac{T}{\alpha S_y}\left\{\frac{1}{r}\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}\right\} - \left\{\int_0^t \frac{\partial s}{\partial t} d\tau\, e^{-\alpha(t-\tau)}\right\} = \frac{S_t \frac{\partial h}{\partial t}}{\alpha S_y}$$

Boulton's equation for unconfined aquifer

$B = \sqrt{\frac{T}{\alpha S_y}}$ = Boulton's coefficient

above solution is complicated so, final answer is ?

no need to remember

$$S(r,t) = \frac{Q}{4\pi T}\int_0^\infty \frac{2}{x}\left[1 - e^{-u}\left(\cosh u_2 + \mu \sinh u_2\right)\right] J_0\left(\frac{rx}{\nu B}\right) dx$$

x: dummy variable

Bessel function of zeroth order

where

$u_1 = 0.5\alpha t\, N(1+x^2)$

$u_2 = 0.5\alpha T S_v$

where $N = \frac{(S_t + S_y)}{S_t}$

$\nu = \sqrt{\frac{N-1}{N}}$

$M = \frac{N(1-x^2)}{S_q}$

$B = \sqrt{\frac{T}{\alpha S_y}}$

$S_q = \sqrt{N^2(1+x^2)^2 - 4Nx^2}$

परेशान नहीं होना है

$-\alpha(t-\tau)$ नहीं याद करना

UPCP

7/3/17

$$\frac{T}{\alpha S_y}\left(\frac{1}{r}\frac{\partial s}{\partial r}+\frac{\partial^2 s}{\partial r^2}\right)-\cancel{\alpha S_y}\int_0^t e^{-\alpha(T-\tau)}\frac{\partial s}{\partial t}\,d\tau=\frac{S_t}{\alpha S_y}\frac{\partial s}{\partial t}$$

$$s=\frac{Q}{4\pi T}\underbrace{W(u)}_{\text{Well }f^n}$$

$$u=\frac{r^2 S_t}{4Tt}$$

$$B=\sqrt{\frac{T}{\alpha S_y}}\quad\text{(dimension of length)}$$

$$B=f(S_y, T, \alpha)$$

Aquifer acts in a different way for diff values of B.

$$s(r,t)=W(r, t, T, S_t, S_y, \alpha)\times\frac{Q}{4\pi T}$$

$$=\frac{Q}{4\pi T}W(\pi_1, \pi_2, \pi_3, \pi_4)$$

$\pi_1, \pi_2, \pi_3, \pi_4$ are non-dimensional parameters involving the prior six.

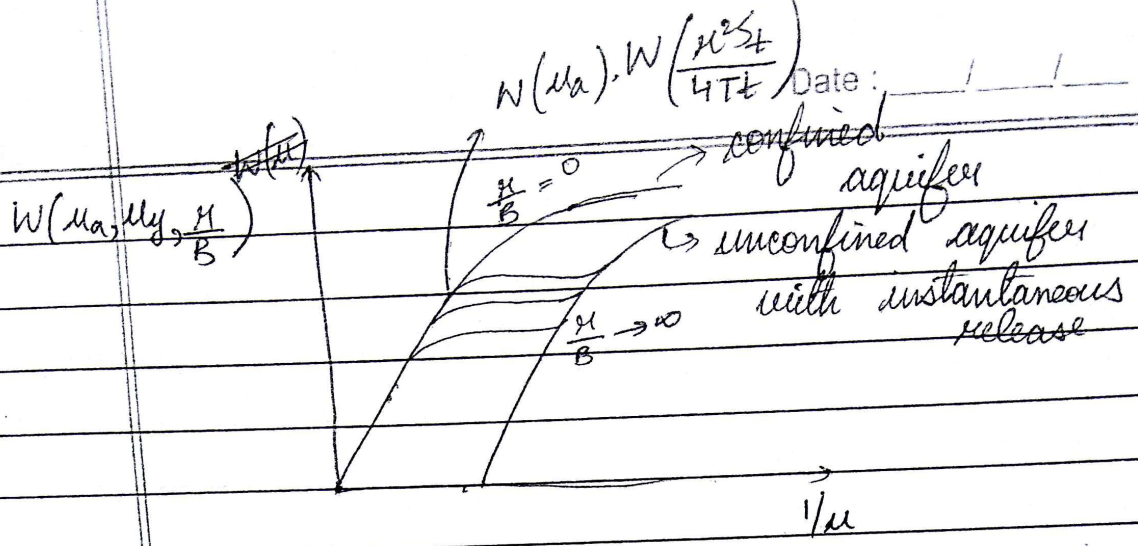
$$\pi_1=\frac{r^2 S_t}{4Tt}$$

$$\pi_2=\frac{r^2 S_y}{4Tt}$$

$$\pi_3=\frac{S_t+S_y}{S_y}\rightarrow\text{almost 1 practically}\ \because S_t \ll S_y$$

$$\pi_4=\frac{r}{B}$$

$$=\frac{Q}{4\pi T}W\left(\underbrace{\frac{r^2 S_t}{4Tt}}_{u_a}, \underbrace{\frac{r^2 S_y}{4Tt}}_{u_y}, \frac{r}{B}\right)$$



$s(r,t) = \frac{Q}{4\pi T} W\left(u_a, u_y, \frac{r}{B}\right)$

As $\frac{r}{B} \to \infty$,

If $\frac{r}{B} = 0 \Rightarrow B \to \infty$

$B = \sqrt{\frac{T}{\alpha S_y}} \to \frac{1}{\alpha} \to \infty$

Acts as a confined aquifer

∴ u_a will govern.

$\Rightarrow \frac{r}{B} \to \infty \Rightarrow \frac{1}{\alpha} \to 0$

u_y will govern.

For intermediate values of $\frac{r}{B}$, it'll first act as confined aquifer and then unconfined.

→ unconfined aquifer, Q, at r and t, drawdown?

given – T, S_t, S_y, α

→ Find $B = \sqrt{\frac{T}{\alpha S_y}}$

$$u_a = \frac{r^2 S_t}{4Tt}$$

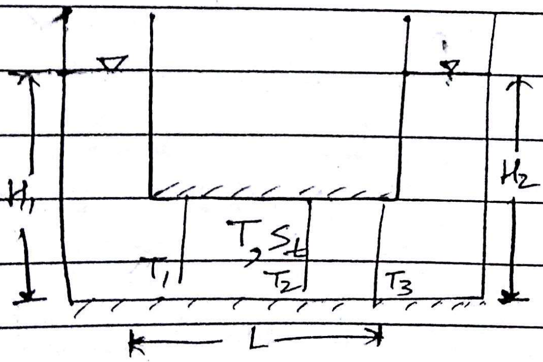
$$u_y = \frac{r^2 S_y}{4Tt}$$

Compute $s = \frac{Q}{4\pi T} W(u)$

For an unconfined aquifer, we'll have a family of curves depending on values of r/B.

* Numerical Methods

1-D flow



$$T\frac{\partial^2 h}{\partial x^2} = S_t\frac{\partial h}{\partial t} \rightarrow \text{governing eq}^n$$

changing T along L.

$T = f(x)$

Convert the differential eq^n into a system of algebraic equations. Solution of that gives us the dependant variable.

Date : ___/___/___

Finite difference uses Taylor series
FEM uses shape functions.

Most of the problems are unsteady state
FEM is used for steady state.
→ use finite difference

$$T\frac{\partial^2 h}{\partial x^2} = S_t \frac{\partial h}{\partial t}$$

Boundary cond^n $\begin{cases} h = H_1 & \text{at } x = 0 \\ h = H_2 & \text{at } x = L \end{cases}$

Divide into a no. of segments

Nodes 1 2 3 4 (i−1) i i+1 . N

L

Δx = length of each segment

Initial cond^n → $t = 0$, $h = h_i$, $0 \le x \le L$

Using Taylor series approximation,

$$h_{i+1} = h_i + \left(\frac{\partial h}{\partial x}\right)_i \Delta x + \underbrace{\frac{1}{2!}\left(\frac{\partial^2 h}{\partial x^2}\right)_i (\Delta x)^2 + \ldots}_{\text{Neglect}}$$

$$h_{i+1} \simeq h_i + \left(\frac{\partial h}{\partial x}\right)_i \Delta x$$

$$\left(\frac{\partial h}{\partial x}\right)_i = \frac{h_{i+1} - h_i}{\Delta x} \rightarrow \text{Forward difference approximation}$$

$$\left(\frac{\partial h}{\partial x}\right)_i = \frac{h_i - h_{i-1}}{\Delta x} \rightarrow \text{Backward difference approximation}$$

Date

$$\left(\frac{\partial h}{\partial x}\right)_i = \frac{h_{i+1} - h_{i-1}}{2\Delta x} \rightarrow \text{central difference approximation}$$

$$h_{i-1} = h_i - \left(\frac{\partial h}{\partial x}\right)_i \Delta x + \underbrace{\frac{1}{2!}\left(\frac{\partial^2 h}{\partial x^2}\right)(\Delta x)^2}_{\text{getting automatically deleted}}$$

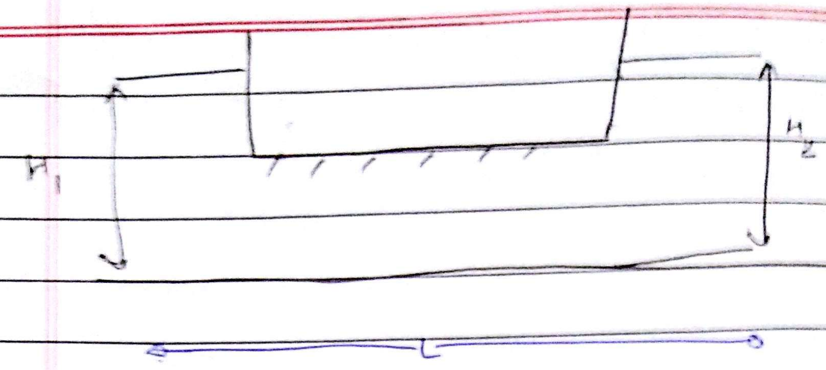
$h_{i-1} - h_{i+1}$ → much better approximation

$$h_{i+1} + h_{i-1} = 2h_i + \frac{2}{2!}\left(\frac{\partial^2 h}{\partial x^2}\right)_i (\Delta x)^2 + \underbrace{\ldots}_{\text{Neglect}}$$

$$\left(\frac{\partial^2 h}{\partial x^2}\right)_i = \frac{h_{i+1} + h_{i-1} - 2h_i}{\cancel{2}(\Delta x)^2}$$

$$\left(\frac{\partial^2 h}{\partial x^2}\right)_i = \frac{h_{i-1} - 2h_i + h_{i+1}}{(\Delta x)^2}$$

$$\rightarrow f_{i+1} = f_i + \Delta x_{i+1} f_i' + \frac{\Delta x_{i+1}^2}{2} f_i'' + \frac{\Delta x_{i+1}^3}{6} f_i'''$$



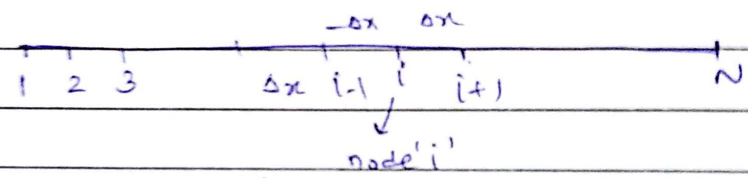
"Using Finite difference Method"

$$T\frac{\partial^2 h}{\partial x^2} = S_t \frac{\partial h}{\partial t}$$

Initial condition $h = h_0$ $0 \le x \le L$, $t = 0$

Boundary condition $h = H$, $x = 0$, $t \ge 0$

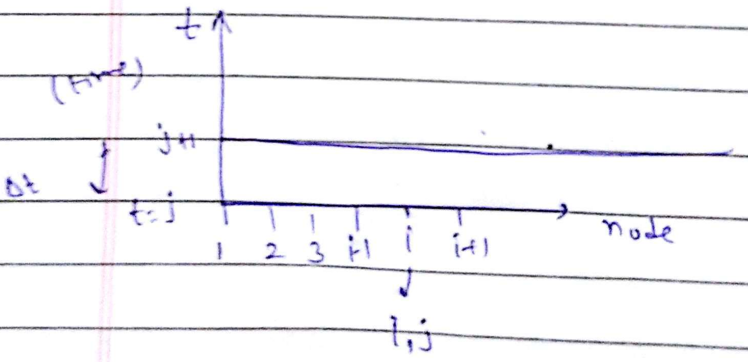
$h = H_2$, $x = L$, $t \ge 0$



$$\left(\frac{\partial h}{\partial x}\right)_i = \frac{h_{i+1} - h_{i-1}}{2\Delta x}$$

at i^{th} node

$$\left(\frac{\partial^2 h}{\partial x^2}\right)_i = \frac{h_{i-1} - 2h_i + h_{i+1}}{(\Delta x)^2}$$



$h_{1,j} = H_1$, $h_{2,j} = h_0$, $h_{3,j} = h_0$, $h_{4,j} = h_0$, --- $h_{N,j} = H_2$

$h_{N-1,j} = h_0$

explicit method

Date: ___/___/___

$$h_{i,j+1} = h_{i,j} + \left(\frac{\partial h}{\partial t}\right)_{i,j} \Delta t$$

using $T \frac{\partial^2 h}{\partial x^2} = S_t \frac{\partial h}{\partial t}$ at i,j

$$\Rightarrow T_{i,j} \left[\frac{h_{i-1,j} - 2h_{i,j} + h_{i+1,j}}{(\Delta x)^2} \right]$$

$$= S_{t\,i,j} \left[\frac{h_{i,j+1} - h_{i,j}}{\Delta t} \right]$$

only one unknown,

$h_{i,j+1}$

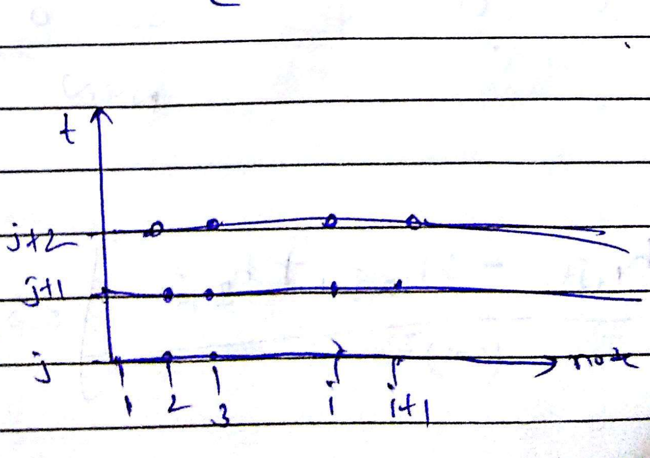
So, $$h_{i,j+1} = \frac{T_{i,j}\,\Delta t}{S_{t\,i,j}} \left[\frac{h_{i-1,j} - 2h_{i,j} + h_{i+1,j}}{(\Delta x)^2} \right] + h_{i,j}$$

at Node, 2

$$T_{2,j} \left[\frac{h_{1,j} - 2h_{2,j} + h_{3,j}}{(\Delta x)^2} \right] = S_{t,2,j} \left[\frac{h_{2,j+1} - h_{2,j}}{\Delta t} \right]$$

$$\Rightarrow T_{2,j} \left[\frac{H_1 - 2h_0 + h_0}{(\Delta x)^2} \right] = S_{t,2,j} \left[\frac{h_{2,j+1} - h_0}{\Delta t} \right]$$

So, $h_{2,j+1}$ = can be calculated from above.



above method is called explicit method

⇒ only one known

Date : ___/___/___

Restriction on Δt

Δt can't be decided independently of Δx.

$\Delta t \leq$ (leave it (5))

Above method is not used prominently.

let's write eqn at $i, j+1$

Implicit Approximation

$$T_{i,j+1}\left[\frac{h_{i-1,j+1} - 2h_{i,j+1} + h_{i+1,j+1}}{(\Delta x)^2}\right] = S_{t_{i,j+1}}\left[\frac{h_{i,j+1} - h_{i,j}}{\Delta t}\right]$$

above eqn 3 unknown

we know $h_{i-1,j}, h_{i,j}, h_{i+1,j}$

$h_{i-1,j+1}, \; h_{i,j+1}, \; h_{i+1,j+1}$

Node →

Node ↓

$$\underbrace{\begin{array}{c|cccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & & N \\
\hline
1 & 1 & 0 & 0 & 0 & 0 & 0 & & \\
2 & 1 & 1 & 1 & 0 & 0 & 0 & - & \\
3 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & & & & 0 & 0 & 0 \\
5 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
6 & & & & & & & & \\
N & 0 & 0 & 0 & & - & - & 0 & 1
\end{array}}_{N \times N \; (H_2)}
\underbrace{\begin{bmatrix} h_{1,j+1} \\ h_{2,j+1} \\ h_{3,j+1} \\ h_{4,j+1} \\ \vdots \\ h_{N,j+1} \end{bmatrix}}_{N \times 1}$$

$S =$

H_1

at Node, 2

$$T_{2,j+1}\left[\frac{h_{1,j+1} - 2h_{2,j+1} + h_{3,j+1}}{(\Delta x)^2}\right] = S_{t_{2,j+1}}\left[\frac{h_{2,j+1} - h_{2,j}}{\Delta t}\right]$$

$$\frac{T_{2,j+1}}{(\Delta x)^2} h_{1,j+1} - \left(\frac{2T_{2,j+1}}{(\Delta x)^2} + \frac{S_{t2,j+1}}{\Delta t}\right) h_{2,j+1}$$

$$+ \frac{T_{2,j+1}}{(\Delta x)^2} h_{3,j+1} = -\left(\frac{S_{t2,j+1}}{\Delta t}\right) h_{2,j}$$

lets make a matrix form

Nodal eq^n (matrix) at Node ②

$$\left[\frac{T_{2,j+1}}{(\Delta x)^2} \;,\; -\left(\frac{2T_{2,j+1}}{(\Delta x)^2} + \frac{S_{t2,j+1}}{\Delta t}\right) \;,\; \frac{T_{2,j+1}}{(\Delta x)^2}\right]_{1\times 3} \begin{bmatrix} h_{1,j+1} \\ h_{2,j+1} \\ h_{3,j+1} \end{bmatrix}_{3\times 1} = \frac{-S_{t2,j+1}}{\Delta t}\left(h_{2,j}\right)$$

So, Matrix G (Global matrix)

Node 2 is bounded by node ① and ③.

So,

$$G_{21} = \frac{T_{2,j+1}}{(\Delta x)^2}$$

$$G_{22} = -\left(\frac{2T_{2,j+1}}{(\Delta x)^2} + \frac{S_{t2,j+1}}{(\Delta x)^2}\right)$$

$$G_{23} = \frac{T_{2,j+1}}{(\Delta x)^2}$$

$$G_{24} = 0$$

$$G_{25} = 0 \quad \vdots \quad G_{2N} = 0$$

→ Known quantity

Date : ___/___/___

$GH = T$

$$T = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ \vdots \\ N \end{array} \begin{bmatrix} H_1 \\ \\ \\ \\ \\ \\ H_2 \end{bmatrix}$$

$T_{11} = H_1$

$T_{21} = -\left[\frac{S_{t2,j+1}}{\Delta t}\right] h_{2,j}$

$T_{31} = -\left[\frac{S_{t3,j+1}}{\Delta t}\right]\left[h_{3,j}\right]$

$T_{N1} = H_2$

Similarly we can write for node (3)

So, $G_{31} = 0$

$G_{32} = \frac{T_{3,j+1}}{(\Delta x)^2}$

$G_{33} =$

$G_{34} = \frac{T_{3,j+1}}{(\Delta x)^2}$

at Node (1), (N) ⇒ Boundary Condition

$h_{1,j+1} = H_1$

So, $G_{11} = 1$, $G_{12} = 0$, ... $G_{1N} = 0$

and $T_{11} = H_1$

Similarly at Node (N)

$h_{N,j+1} = H_2$, $G_{N1} = 0$, $G_{N2} = 0$ ——

$G_{N-1,N} = 0$, $G_{NN} = 1$

14/3/17

vector of unknown nodal points

Date ___/___/___

$$GH = T$$

Coefficient Matrix

RHS vector

$$\boxed{H = G^{-1} T}$$

Coefficient Matrix does not change with time.

advantage is: Δt and Δx are independent

Above method is called implicit method.

* Most of the Software is based on implicit method.

G = Tridiagonal matrix.

(diagonal element as two off diagonal elements)

14/3/17 $T\left(\frac{\partial^2 h}{\partial x^2}\right) = S_t \frac{\partial h}{\partial t}$

Initial Condition $h = h_0$, $t = 0$, $0 \le x \le L$

Boundary Condition $h = H_1$, $t \ge 0$, $x = 0$

$h = H_2$ $t \ge 0$, $x = L$

$$\begin{array}{c c}
 & \begin{array}{cccccc} 1 & 2 & 3 & 4 & - & N \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ \vdots \\ N \end{array} &
\begin{bmatrix}
1 & 0 & 0 & 0 & - & - \\
a_2 & b_2 & c_2 & & & \\
0 & a_3 & b_3 & c_2 & & \\
0 & 0 & a_4 & b_4 & c_4 & \\
 & & & & & \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}_{N \times N}
\end{array}
\begin{bmatrix} h_1^{j+1} \\ h_2^{j+1} \\ | \\ \\ h_N^{j+1} \end{bmatrix}_{N \times 1}
=
\begin{bmatrix} D_1 \\ D_2 \\ | \\ | \\ D_N \end{bmatrix}_{N \times 1}$$

$$AH = D \Rightarrow \boxed{H = A^{-1} D}$$

UPCP

Date : / /

Solution of tri-diagonal system equations
(Thoms Algorithm)

Tri-diagonal matrix

$$\underbrace{\begin{bmatrix} b_1 & c_1 & - & - \\ a_2 & b_2 & c_2 & - \\ 0 & a_3 & b_3 & c_3 - \\ = - & a_{n-1} & b_{n-1} & c_{n-1} \\ - & - & a_n & b_n \end{bmatrix}}_{(n\times n)}^{A} \underbrace{\begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ \vdots \\ H_n \end{bmatrix}}_{H} = \underbrace{\begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ \vdots \\ D_n \end{bmatrix}}_{D}$$

$AH = D$

Coefficient Matrix $= A = LU$ = Product of two Matrix (decomposition of A matrix)

lower Triangular Matrix $(n\times n)$

upper triangular Matrix $(n\times n)$

$LUH = D$

$LY = D$

$Y = UH \Rightarrow L^{-1}D$

$\boxed{H = U^{-1}L^{-1}D}$ $\boxed{H = U^{-1}Y}$

$$\underbrace{\begin{bmatrix} b_1 \\ a_2 & b_2 & c_2 \\ 0 & a_3 & b_3 & c_3 \\ - \\ - \\ - & - & a_{n-1} & b_{n-1} & c_{n-1} \\ - & & & a_n & b_n \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} \alpha_1 & 0 & 0 & - \\ a_2 & \alpha_2 & 0 & - \\ 0 & a_3 & \alpha_3 & - \\ 0 & 0 & a_4 & \alpha_4 \\ & & & & \ddots \\ & & & & a_n & \alpha_n \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 1 & \beta_1 & 0 & 0 & 0 \\ 0 & 1 & \beta_2 & - & - \\ 0 & 0 & 1 & \beta_3 \\ 0 & 0 & 0 & 1 & \beta_4 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{U}$$

Date: ___/___/___

$$\boxed{\begin{aligned} &\alpha_1 = b_1 \quad \Rightarrow \\ &\beta_i = \frac{c_i}{\alpha_i}, \; i \in (1, n-1) \\ &\alpha_i = b_i - a_i \beta_{i-1} \; ; \; i \in (2, n) \end{aligned}}$$

$LUH = D$ $\quad \boxed{UH = y}$

$Ly = D$

$$\begin{bmatrix} \alpha_1 & 0 & \cdots \\ a_2 & \alpha_2 & \cdots \\ 0 & a_3 & \alpha_3 & \cdots \\ & & & \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ | \\ | \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ | \\ \end{bmatrix}$$

$y_1 = \frac{D_1}{\alpha_1}$

Forward Substitution

$a_2 y_1 + \alpha_2 y_2 = D_2$

$\Rightarrow \frac{a_2 D_1}{\alpha_1} + \alpha_2 y_2 = D_2$

$$\boxed{y_2 = \frac{\alpha_1 D_2 - a_2 D_1}{\alpha_1 \alpha_2}}$$

$$\text{Sly} \quad \boxed{y_i = \frac{D_i - a_i y_{i-1}}{\alpha_i}} \quad i \in (2, n)$$

$y = UH$

$$\begin{bmatrix} 1 & \beta_1 & 0 & \cdots & \cdots \\ 0 & 1 & \beta_2 & \cdots & \cdots \\ 0 & 0 & 1 & \beta_3 & \cdots \\ 0 & 0 & 0 & 1 & \beta_{n-1} \\ & & & & 1 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ | \\ H_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ | \\ y_n \end{bmatrix}$$

$H_n = y_n \qquad 1 \times H_{n-1} + \beta_{n-1} H_n = y_{n-1}$

Date : ___/___/___

$H_n = y_n$

$\boxed{H_i = y_i - \beta_i h_{i+1}}$ $i \in (n-1, 1)$

Backward Substitution

Now,

$$\begin{bmatrix} b_1 & c_1 & & \\ a_2 & b_2 & c_2 & \\ 0 & a_3 & b_3 & c_3 \\ & & & \end{bmatrix}$$

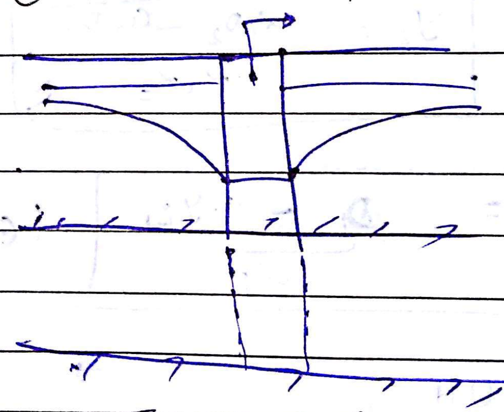
A

Step I from Matrix "A" get compute α_i and β_i for ...

Step 2 than y_i, than β_i

Step III Calculate ~~H/H~~ H_i

In practice we encounter two types of Problem
(A) Forward Problem or Direct Problem
(B) Backward Problem or inverse Problem



Confined

we need to know T, S_t for Calculating drawdown
"we already know the
$S(r,t) = \frac{Q}{4\pi T} W(u)$

These problems are direct Problem

In the field, pumping test for aquifer property

t	S_o → Observed drawdown at well
	$S_o(r,t)$ observed

Now How you will calculate T, S_t using t v/s S curve

this is indirect problem

(UPCP)

Date : ___/___/___

Assume T, S_t

then Calculate $S_p(r,t)$
↓ Predicted value using assume T, S_t

$S_p(r,t)$ को बराबर बनाना है $S_o(r,t)$ के (i)

$S_p - S_o$ = deviation.

$$Z = \sum_{i=1}^{n} (S_{p_i} - S_{o_i})^2$$ = Sum of Square of all deviation b/w Predicted and observed drawdown)

I have to minimize the 'Z' (i.e. least square ~~area~~ error)

$Z = f(T, S_t)$ ⇒ $\frac{\partial Z}{\partial T} = 0$ and $\frac{\partial Z}{\partial S_t} = 0$

So, minimize the Z w.r.t T, S_t

So, above Problem is called inverse Problem or Backward Problem.

Above Procedure is iterative optimization Procedure.

15/3/17 Pumping Tests:- [Indirect Problems]

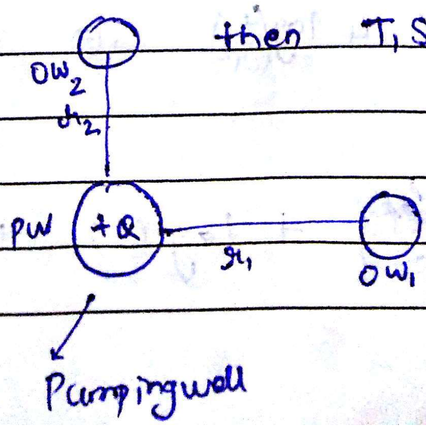
① – Direct Problem / Forward Problem / Analysis Problem
→ for Given T, S_t, Q
$S = ??$

② Inverse Problem / Backward Problem / Estimation Problem

Given, Q, $S(r,t)$

then T, S_t = ?? (They are estimates not true value)

Pumping Test



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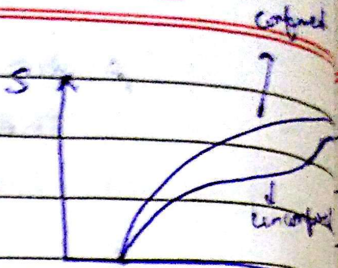
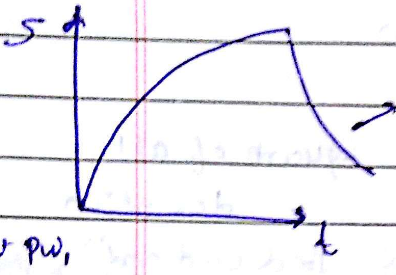
at $t=0$, $h=h_0$

for $PW_1 \longrightarrow S(r,t)$ at OW_1

OW_1

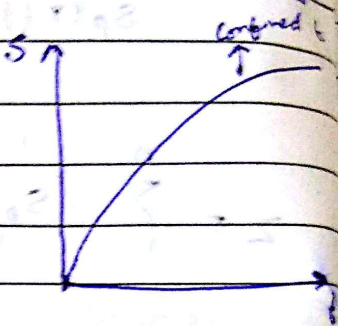
OW_2



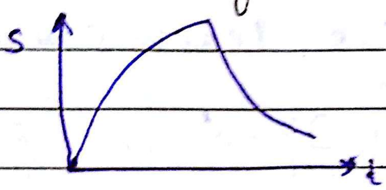


at pw_1

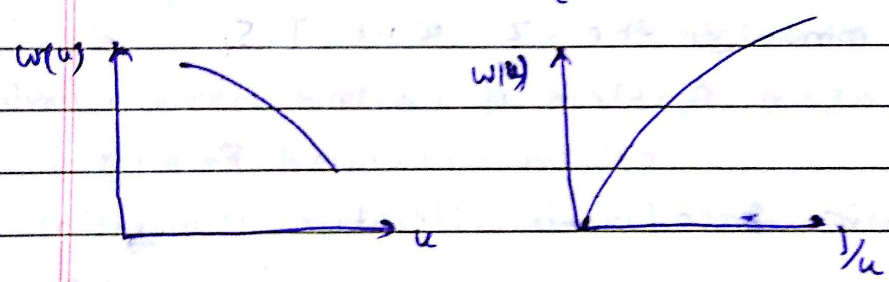
at pw_1



let's take confined aquifer



Type curve



$u = \frac{r^2 S_t}{4Tt}$

Estimation of confined aquifer Parameters:

(T, S_t)

$S = \frac{Q}{4\pi T} W(u)$ $\quad u = \frac{r^2 S_t}{4Tt} \Rightarrow \frac{1}{u} = \frac{4Tt}{r^2 S_t}$

$\log s = \log \frac{Q}{4\pi T} + \log W(u) \rightarrow ①$

$\log\left(\frac{1}{u}\right) = \log\left(\frac{4T}{S_t}\right) + \log\left(\frac{t}{r^2}\right) \rightarrow ②$

from ①

$\log W(u) = \log\left(\frac{4\pi T}{Q}\right) + \log(s) \rightarrow ③$

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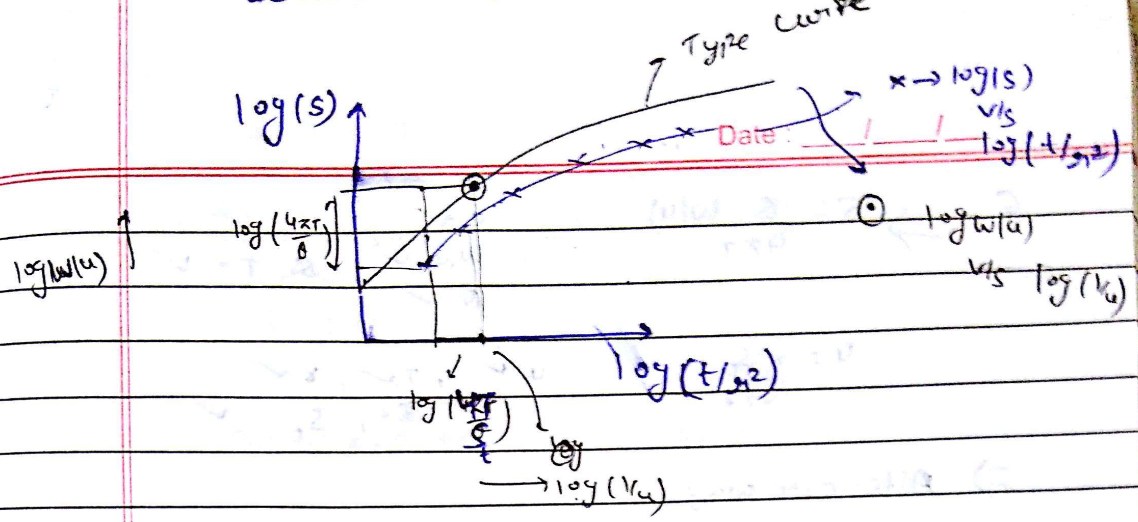
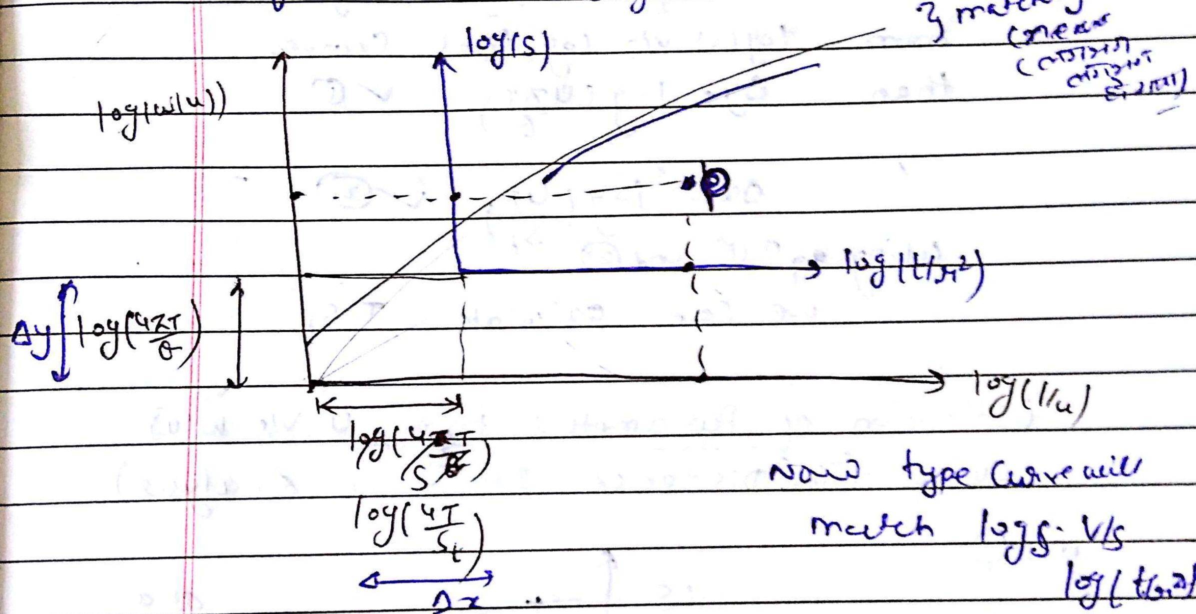
we have data of S v/s t



we have to shift the curve origin



Now type curve will match logs V/s log(t/r²) curve

Type curve matching Procedure

1. plot $\log(t/r^2)$ V/s $\log(s)$
2. Draw on a tracing sheet $\log(1/u)$ V/s $\log W(u)$
   "Scale should be same"
3. Put the tracing sheet on $\log(t/r^2)$ V/s $\log(s)$ sheet. [Superimposing type curve over $\log(t/r^2)$ V/s $\log(s)$.
4. Keeping the axis parallel try to match the both curve as closely as possible.
5. Consider an any arbitary point 'P' and get $\frac{t}{r^2}$, S from $\log S$ V/s $\log(t/r^2)$
   and $\frac{1}{u}$ and $W(u)$ from type curve

UPCP

Date: / /

6. $\Rightarrow S = \frac{Q}{4\pi T} W(u)$ S ✓ W(u) ✓ Q ✓ So, T = ✓

$u = \frac{r^2 S_t}{4Tt}$ u ✓, T ✓, r ✓ So, S_t = ✓.

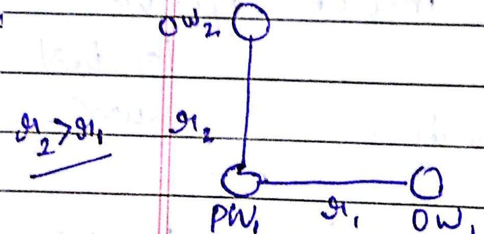
$\Rightarrow$ Alternate way,

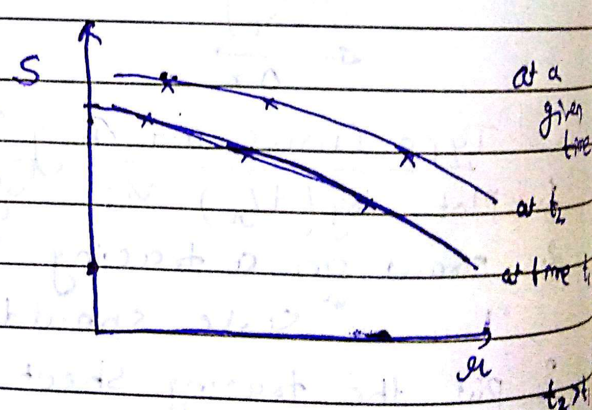
Calculate the offset of the type curve from log(s) v/s log(t/r^2) curve, than $\Delta y = \log\left(\frac{4\pi T}{Q}\right)$ ✓ ①

$\Delta x = \log\left(\frac{4T}{S_t}\right)$ ✓ ②

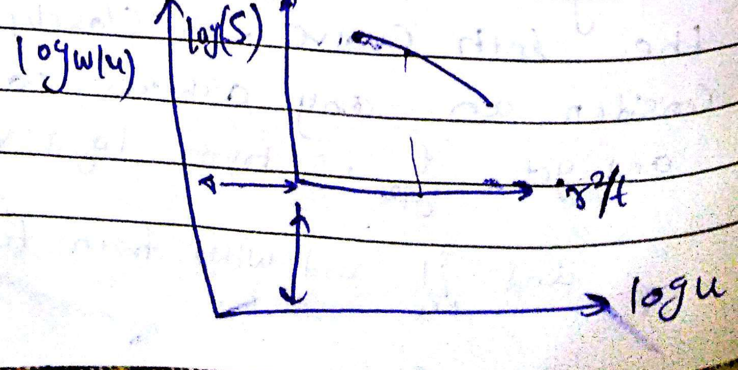
using eq^n ① and ② we can calculate T, S_t

Estimation of Parameters from u v/s W(u) Curves :- (Time Distance Drawdown Analysis)





to analyze above curve we will use W(u) v/s u type curve.



Calculate shift you can calculate

Estimation using Cooper-Jacob Eqn

$u < 0.05$

$$S = \frac{2.303Q}{4\pi T}\log\left(\frac{2.25 T t}{r^2 S_t}\right)$$

above eqn eleminate use of Type Curve.

$$S = \frac{2.303Q}{4\pi T}\log\left(\frac{2.25 T}{r^2 S_t}\right) + \frac{2.303Q}{4\pi T}\log t$$

S v/s log t

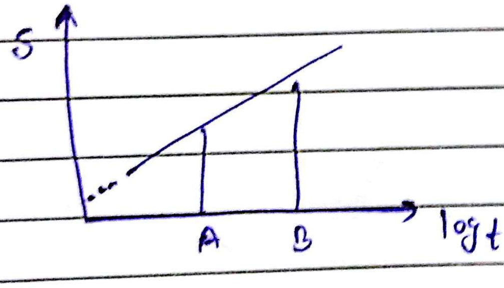
$$Y = B + AX$$

$$A = \frac{2.303Q}{4\pi T}$$

$$B = \frac{2.303Q}{4\pi T}\log\left(\frac{2.25 T}{r^2 S_t}\right)$$

Another way

Plot S v/s log(t)



$$S_A = \frac{2.303Q}{4\pi T}\log\left(\frac{2.25 T t_A}{r^2 S_t}\right)$$

$$S_B = \frac{2.303Q}{4\pi T}\log\left(\frac{2.25 T t_B}{r^2 S_t}\right)$$

$$\boxed{S_A - S_B = \frac{2.303Q}{4\pi T}\log\left(\frac{t_A}{t_B}\right)}$$

t_A and t_B are one log cycle apart

so $\frac{t_A}{t_B} = 10$

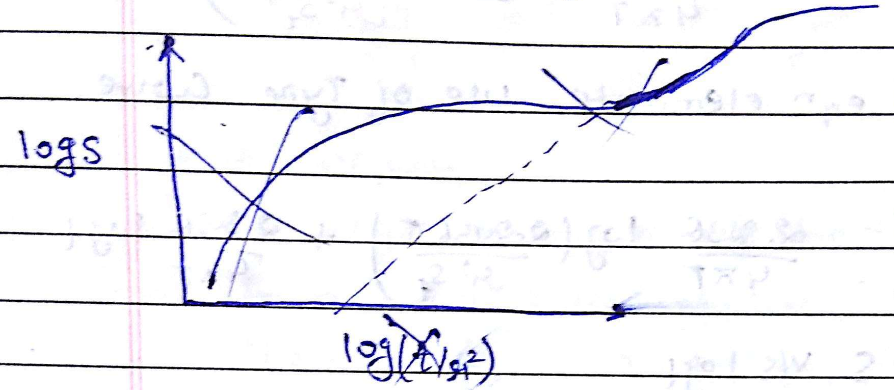
$$\boxed{S_A - S_B = \frac{2.303Q}{4\pi T}}$$

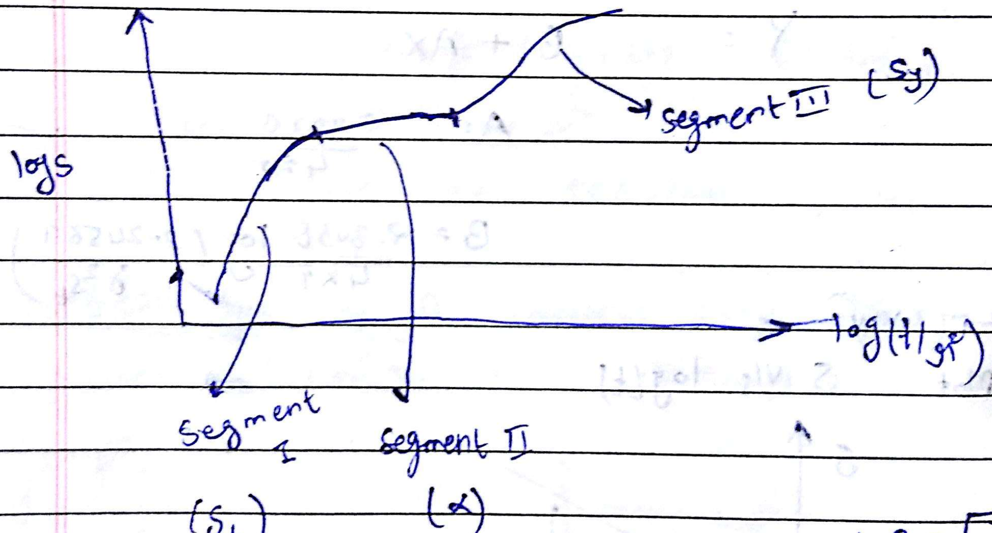
22/3/17

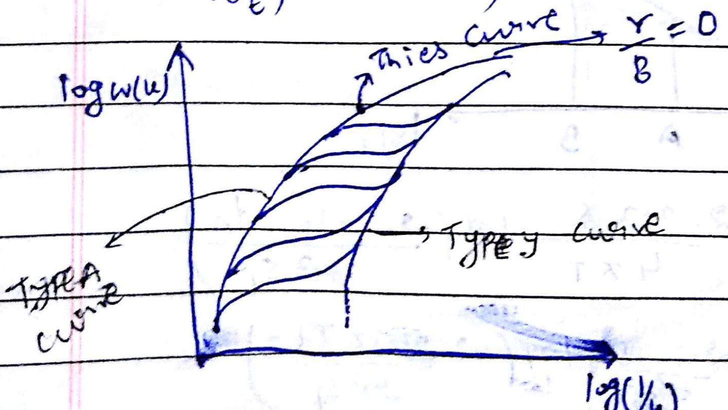
Date : ___/___/___

Estimation of Unconfined Aquifer Parameters from pumping Test data

$T, S_t, S_y, \alpha \rightarrow$ Parameters



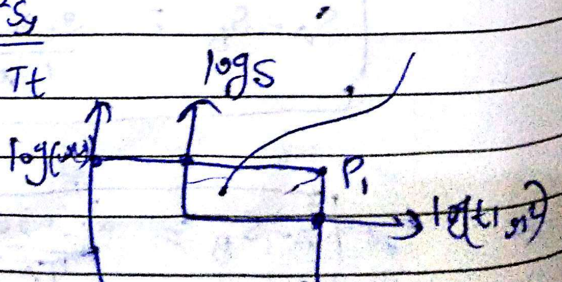




$\Rightarrow B = \sqrt{\frac{T}{\alpha S_y}}$

For different $\frac{r}{B}$ values

$u_A = \frac{r^2 S_t}{4Tt}$, $u_y = \frac{r^2 S_y}{4Tt}$



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Steps :-

① Match as much as small time data as possible with Type A curve.

② At the matched position get $S, \frac{t}{r^2}$ from $W(u_A), \frac{1}{u_A}, \frac{r}{B}$

③ $S = \frac{Q}{4\pi T} W(u_A) \rightarrow$ get T

④ $u_A = \frac{r^2 S_t}{4Tt}$ get S_t

(above steps are same as confined aquifer)

⑤ Try to match the larger time data with type 'y' curve, for the same $\left(\frac{r}{B}\right)$ value wala curve obtained from step ②.

⑥ now get $\frac{t}{r^2}$, S, $W(u_y)$ and $\left(\frac{1}{u_y}\right)$

$S = \frac{Q}{4\pi T} W(u_y)$. Solve for T,

and $\frac{r^2 S_y}{4Tt} = u_y$ get S_y.

If 'T' value which you obtained from step ③ and step ⑥ should be same. otherwise go for new trials.

⑦ segment II corresponds to $\frac{r}{B}$ value

from $\frac{r}{B}$ we can calculate α.

$r\sqrt{\frac{\alpha S}{T}}$ $r\sqrt{\frac{\alpha S}{B}}$ = —

$\frac{r}{B}$ Value

(t/r^2)

$\log(1/u)$

(UPCP)

Date: __/__/__

Modflow = software used for solving above Problems

$$Z = (S_0^i - S_p i)^2$$

observed → predicted

min (Z) and calculate T, S_t, S_y, α

11 Regional Ground Water Flow Analysis :-

1. Ground water Developement
   - during Non Monsoon season decline in water table.
   - Monsoon ⇒ increase in WT.
2. Causes for over exploitation
3. Water logging in Canal Command Areas.
4. Ground water Contamination
5. Conjuctive use (Combination of Surface and ground water)

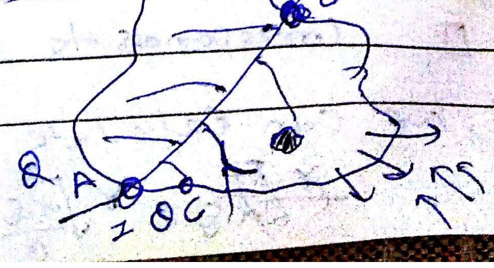
Two types of approach
1. Integral water Balance approach
2. Distributed Modelling Approach

water Balance eqn
Inflow − outflow = Storage

Soln of 2-D ground water eqn.

27/3/17 Integral Balance Approach :-

$B = \theta_C - \theta_A - \theta_B$



Inflow − outflow = Change in Storage

Inflows

1. Recharge from Rainfall } R_f
2. Recharge from Irrigation }
3. Recharge from Surface Sources } R_s

(Lakes, Ponds, Effluent River, Canals)

Outflows:-

1. Base flow (during non Monsoon season) – B
2. Pumping (Draft) – D
3. outflow from the Region – O
4. Ground Evaporation – Neglect (very low)

$$\Rightarrow \quad R_f + R_s - (B + D + O) = S_g = \text{Change in Storage}$$

assume S_t to be very low (Because ET Regional level पर होता रहे)

So, $S_g = S_y (\Delta h) A$ → Area of Region.

Δh → change in water Table

So, $R_f + R_s - (B + D + O) = S_y \Delta h A = S_g = ??$

Direct Problem

$R_f = F_f P$

P = Rainfall

F_f = Recharge Coefficient

→ depends on Land use and Type of soil.

R_s = Use Darcy's law

B = Base flow = Measure stream flow at inlets and outlet of the Region

("Discharge at gauging sites")

$B = Q_C - (Q_A + Q_B)$

D - Well Census

O - Use Darcy law at the Boundary of the Region

Use WW Contours and 'T'

Now Solve for Δh = ??

for solving above Problem i need to know aquifer Parameter.

Inverse Problem

Given { Rainfall — River Gauge Discharge Data
Well Census — Water Table Data
Pumping Test Data (T) }

~~From these~~ ~~to~~ ~~Predict~~ estimate the best possible value of S_y = ??

Inverse Problem {
what is overall average S_y value = ??
what is recharge coefficient F_f = ? }

during Non Monsoon Season = $R_f \Rightarrow 0$

Oct-May $\quad \cancel{R_f} + R_s - (B+D+O) = S_y \Delta h A$
$\quad$ 0.

Know, S_y = ✓✓

for Monsoon Data

June-Sept $\quad R_f + R_s - (B+D+O) = S_y \Delta h A$

$R_f = F_f P$ $\quad$ From Here F_f = ✓✓

If F_f is low, then surface Runoff is High, so we can take measures accordingly.

* This approach is very simple to use.

* integral Balance gives you the average value over whole area.
* It does not tells about locally.

Date: __/__/__

Distributed Modelling Approach :-

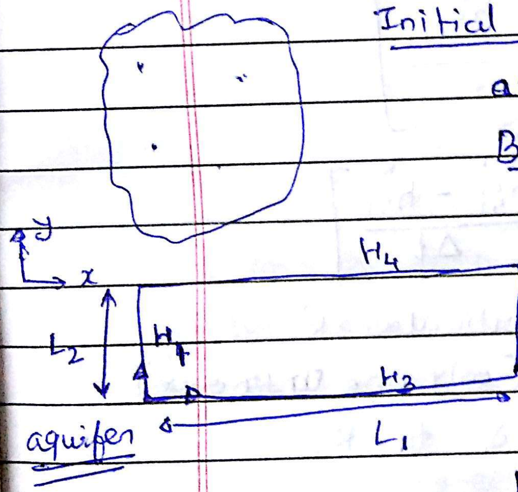
$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y}\right) + \underset{\text{Source}}{Q_R} - \underset{\text{Sink}}{Q_p} = S_y \frac{\partial h}{\partial t}$$

2nd order in x and y
1st order in time

$h(x, y, t) = \checkmark\checkmark$??

31/3/17

$$T_x \frac{\partial^2 h}{\partial x^2} + T_y \frac{\partial^2 h}{\partial y^2} + Q_R - Q_p = S_t \frac{\partial h}{\partial t}$$



Initial Condition

at $t = 0$, $h = h_0$ $\forall\ 0 \le x \le L_1,\ 0 \le y \le L_2$

Boundary Condition

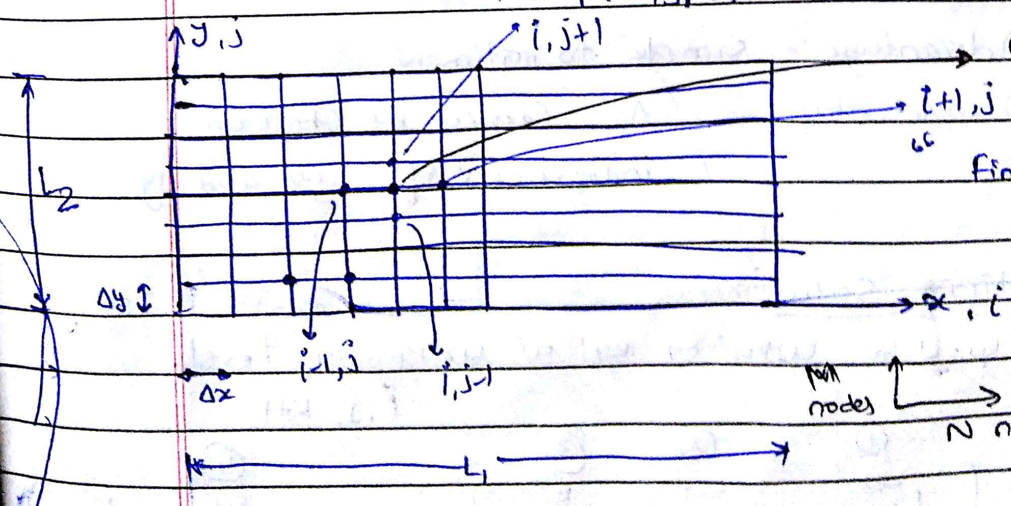
$t > 0$, $h = H_1$ $\forall\ x = 0,\ 0 \le y \le L_2$

$t > 0$ $h = H_2$ $\forall\ x = L_1,\ 0 \le y \le L_2$

$t > 0$ $h = H_3$ $\forall\ 0 \le x \le L_1,\ y = 0$

$t > 0$ $h = H_4$ $\forall\ 0 \le x \le L_1,\ y = L_2$

$h = f(x, y, t)$



"Finite Difference Method"

we will use finite difference at each nodes.

Date : / /

explicit solution

i → index for x
j → index for y
k → index for time (t)
Δt = time step

$\left(\frac{\partial^2 h}{\partial x^2}\right)$ at i,j

at node i, j at Particular k. [writing eqn at known level i, j, k.

$$T_{x_{i,j}} \left[\frac{h^k_{i-1,j} - 2h^k_{i,j} + h^k_{i+1,j}}{(\Delta x)^2} \right]$$

$$+ T_{y_{i,j}} \left[\frac{h^k_{i,j-1} - 2h^k_{i,j} + h^k_{i,j+1}}{(\Delta y)^2} \right]$$

$$+ Q_{R_i} - Q_{P_i} = S_{t_{i,j}} \left[\frac{h^{k+1}_{i,j} - h^k_{i,j}}{\Delta t} \right]$$

we know all values at Particular 'k'.

unknown, $h^{k+1}_{i,j}$ (only one unknown)

Solve, starting from k=0, to k.

Problem

Advantage :- simple to impliment.

Disadvantage :- Δt can not be choosen independent of Δx and Δy.

Implicit Solution

we will be writing eqn at unknown prime level.

i, j, k+1

$$T_{x_{i,j}} \left[\frac{h^{k+1}_{i-1,j} - 2h^{k+1}_{i,j} + h^{k}_{i+1,j}}{(\Delta x)^2} \right] + T_{y_{i,j}} \left[\frac{h^{k+1}_{i,j-1} - 2h^{k+1}_{i,j} + h^{k+1}_{i,j+1}}{(\Delta y)^2} \right]$$

$$Q_{R_i} - Q_{P_i} = S_{t_{i,j}} \left[\frac{h^{k+1}_{i,j} - h^k_{i,j}}{\Delta t} \right]$$

Date : / /

total 5 unknown (5) unknowns

so. Matrix will form

$$\begin{bmatrix} \frac{T_{x_{i,j}}}{(\Delta x)^2} & \frac{T_{x_{i,j}}}{(\Delta x)^2} & \frac{-2}{(\Delta x)^2}\left[\frac{T_{x_{ij}}}{(\Delta x)^2}+\frac{T_{y_{i,j}}}{(\Delta y)^2} - \frac{S_{t_{i,j}}}{\Delta t}\right] & \frac{T_{y_{i,j}}}{(\Delta y)^2} & \frac{T_{y_{i,j}}}{(\Delta y)^2} \end{bmatrix} \begin{bmatrix} h^{k+1}_{i-1,j} \\ h^{k+1}_{i+1,j} \\ h^{k+1}_{i,j} \\ h^{k+1}_{i,j-1} \\ h^{k+1}_{i,j+1} \end{bmatrix}$$

$[1\times5]\ [5\times1]$

nodal matrix

$$= \left[-\frac{S_{t_{i,j}}\, h^{k}_{i,j}}{\Delta t} + Q_{p_i} - Q_{R_i}\right]$$

one nodal matrix corresponding to one node.
we need to assemble $M\times N$ nodal matrix
to form a Global matrix

$$AH = R$$

A → Global matrix $(MN \times MN)$

H → unknown nodal heads → $(MN\times1)$

R → $(MN\times1)$

$$\boxed{h = A^{-1}R}$$

h is Calculated at $t = k+1$
So we can Calculate at other time

$2h^{k+1}_{i,j}$
$+ h^{k+1}_{i,j+1}$
$y)^2$ (5)
h^k

Date : ___/___/___

* If I write $\frac{\partial^2 h}{\partial x^2}$ at node time level K

and $\frac{\partial^2 h}{\partial y^2}$ at node unknown time level $K+1$

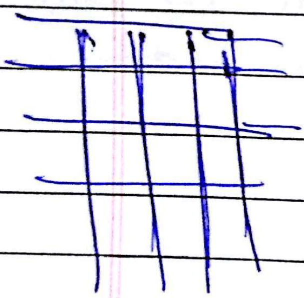
$$T_{x_{i,j}}\left[\frac{h^{K}_{i-1,j} - 2h^{K}_{i,j} + h^{K}_{i+1,j}}{(\Delta x)^2}\right]$$

(1) (2) (3)

$$+ T_{y_{i,j}}\left[\frac{h^{K+1}_{i,j-1} - 2h^{K+1}_{i,j} + h^{K+1}_{i,j+1}}{(\Delta y)^2}\right]$$

$$+ Q_{R_i} - Q_{P_j} = S_{t_{i,j}}\left[\frac{h^{K+1}_{i,j} - h^{K}_{i,j}}{\Delta t}\right]$$

Total unknown = 3 (3 unknown)



we are solving column by column so we can use Tridiagonal nature of matrix.

For above system to be stable

$t=0,\ t=\Delta t,\ t=2\Delta t$

Write $\frac{\partial^2 h}{\partial x^2}$ at $K+1$, and $\frac{\partial^2 h}{\partial y^2}$ $(K+2)$ at

$t=0,\ t=\Delta t$		$t=\Delta t$ to $2\Delta t$		$t=2\Delta t$ to $3\Delta t$	
Column wise solving $K \to K+1$		Row wise $K+1 \to K+2$		Column wise $K+2 \to K+3$	
$\frac{\partial^2 h}{\partial x^2}$	$\frac{\partial^2 h}{\partial y^2}$	$\frac{\partial^2 h}{\partial x^2}$	$\frac{\partial^2 h}{\partial y^2}$	$\frac{\partial^2 h}{\partial x^2}$	$\frac{\partial^2 h}{\partial y^2}$
↓	↓	↓	↓	↓	↓
Known time level K	Unknown time level $K+1$	Unknown time level $K+2$	Known time level $K+1$	Known time level $K+2$	Unknown time level $K+3$

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Date : ___/___/___

Above method is known as

Alternating ~~Alternating~~ Implicit ~~or~~ Explicit
Direction
Method

(ADIE) = Method.

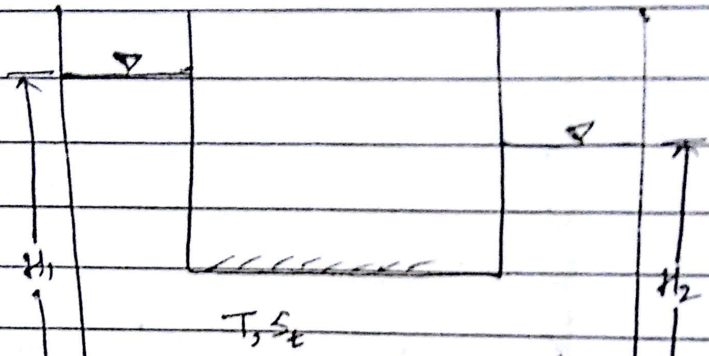
Most of the software used above method for solving Real/~~life problem~~ Problems.

$\Rightarrow$ "What is happening at local nodal level, we can calculate values ~~at~~ locally"

5/4/17

$T\frac{\partial^2 h}{\partial x^2}$

→ CONFINED AQUIFER



SPECIFIED HEAD BOUNDARY

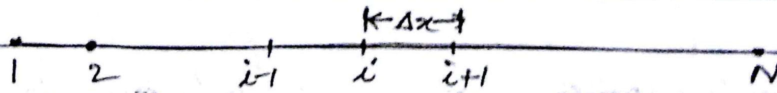
$$T\frac{\partial^2 h}{\partial x^2} + Q_R - Q_P = S_t\frac{\partial h}{\partial t}$$

I.C.

$t=0,\ h=h_0,\ 0 \le x \le L$

$t \ge 0,\ h=H_1,\ x=0$

$t \ge 0,\ h=H_2,\ x=L$



IMPLICIT SCHEME

$$T_i\left[\frac{h_{i-1}^{j+1} - 2h_i^{j+1} + h_{i+1}^{j+1}}{(\Delta x)^2}\right] = S_{t_i}\left[\frac{h_i^{j+1} - h_i^{j}}{\Delta t}\right]$$

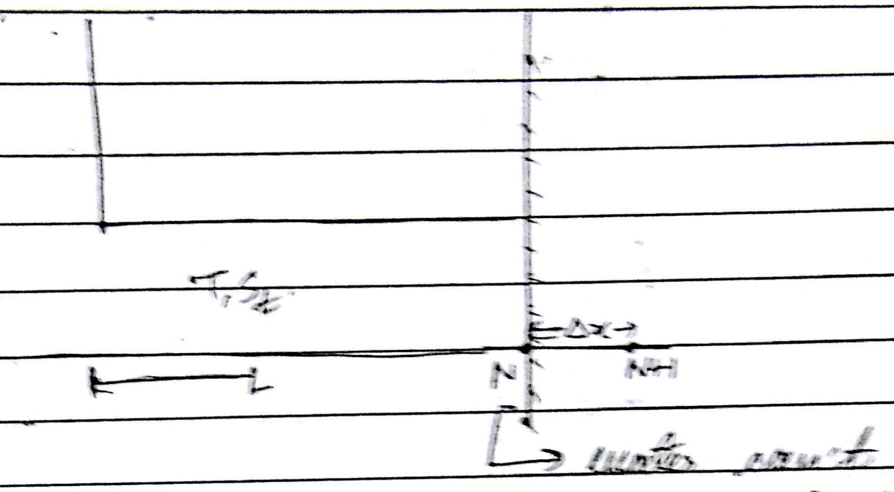
$Ah = D$

Date: __/__/__

$$\begin{bmatrix} 1 & 0 & & \\ & \ddots & & \\ & & & \\ & & & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ \vdots \\ h_N \end{bmatrix} = \begin{bmatrix} h_L \\ \vdots \\ H_R \end{bmatrix}$$

2.

*



$t \geq 0, \quad \frac{\partial h}{\partial x} = 0, \quad x = L$

→ Modify last row to take into account this condition.

→ go to a node beyond N.

(N+1) is out of domain.

Write finite difference equation for the N^{th} node, considering it an interior node.

$$T_N \left[\frac{h_{N-1}^{j+1} - 2h_N^{j+1} + h_{N+1}^{j+1}}{(\Delta x)^2} \right] = S_N \left[\frac{h_N^{j+1} - h_N^{j}}{(\Delta t)} \right] \quad \text{— ①}$$

We've to eliminate the imaginary term h_{N+1}^{j+1}.

(UPCP)

Date : / /

$$\frac{\partial h}{\partial x} = 0$$

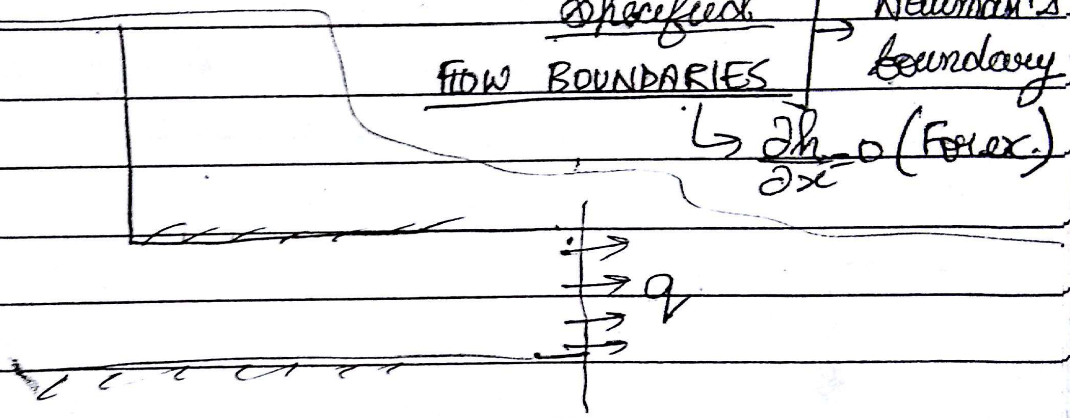
$$\Rightarrow \frac{h_{N+1}^{j+1} - h_{N-1}^{j+1}}{2\Delta x} = 0$$

$$h_{N+1}^{j+1} = h_{N-1}^{j+1} \quad \text{— ②}$$

Eliminate h_{N+1}^{j+1} from equations ① and ②.

* Specified FLOW BOUNDARIES → Neuman's boundary

↳ $\frac{\partial h}{\partial x} = 0$ (For ex.)



Condition

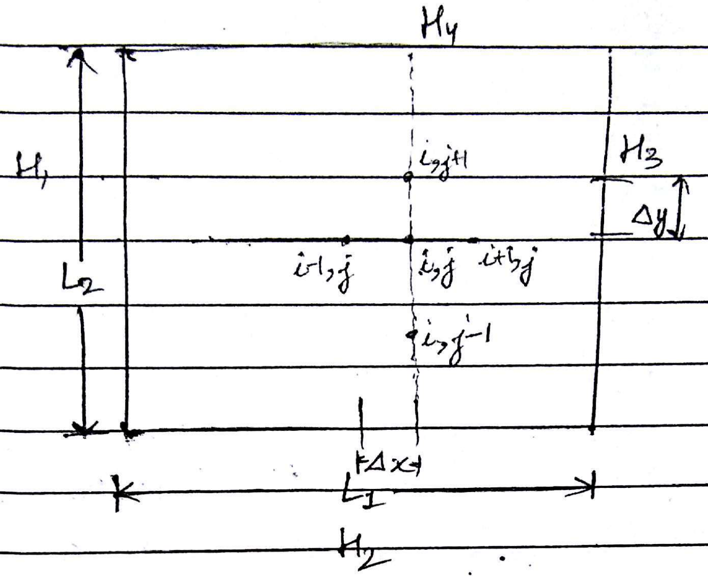
$$-T\frac{\partial h}{\partial x} = q$$

$$\Rightarrow -T_N\left(\frac{h_{N+1}^{j+1} - h_{N-1}}{2\Delta x}\right) = q$$

Basically, eliminate the imaginary node.

Date : ___/___/___

$$T_x \frac{\partial^2 h}{\partial x^2} + T_y \frac{\partial^2 h}{\partial y^2} + Q_R - Q_P = S_t \frac{\partial h}{\partial t}$$



At $t=0$, $h=h_0$, $0 \leq x \leq L_1$, $0 \leq y \leq L_2$

At $t \geq 0$, $h = H_1$, $x=0$, $0 \leq y \leq L_2$

$h = H_3$, $x = L_1$, $0 \leq y \leq L_2$

$h = H_2$, $0 \leq x \leq L_1$, $y=0$

$h = H_4$, $0 \leq x \leq L_1$, $y = L_2$

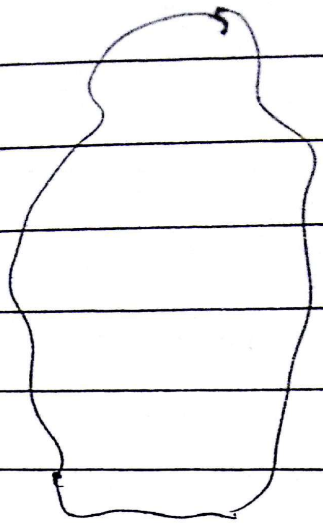
$$T_{x_{i,j}} \left[\frac{h_{i-1,j}^{k+1} - 2h_{i,j}^{k+1} + h_{i+1,j}^{k+1}}{(\Delta x)^2} \right] + T_{y_{i,j}} \left[\frac{h_{i,j-1}^{k+1} - 2h_{i,j}^{k+1} + h_{i,j+1}}{(\Delta y)^2} \right]$$

$$+ Q_{R_i} - Q_{P_i} = S_{t_{i,j}} \left[\frac{h_{i,j}^{k+1} - h_{i,j}^{k}}{\Delta t} \right]$$

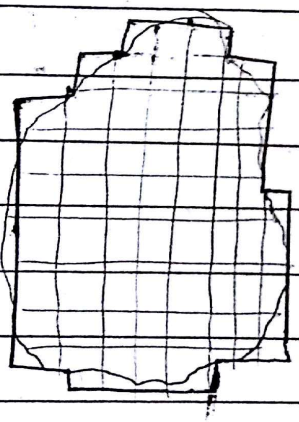
terms having a superscript of $(k+1)$ are unknown.

Practically, aquifers don't have regular shapes like this one (rectangular)

*



We superimpose a finite difference grid.



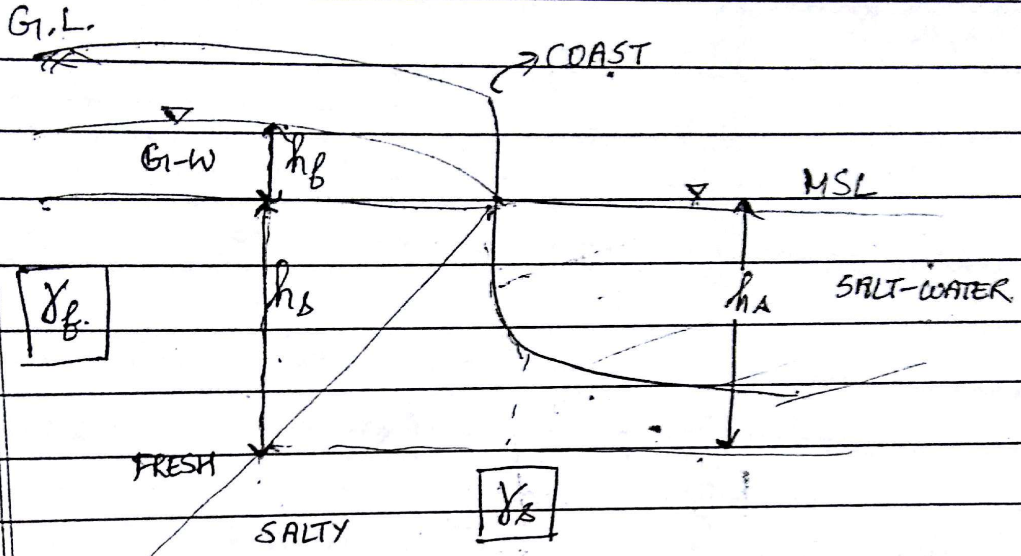
Divide in different parts
We may have 10 ~~p~~ nodes in one dirn
& 20 in another.
Can't solve with calculator → Use coding.

For each row → how many columns

This completes the regional groundwater problem.

Date : ___/___/___

* If aquifers are nearby sea-water, when we pump, sea-water can infiltrate. Pumped out water becomes useless for drinking, irrigation etc.



↳ Interface of salt-water & fresh-water

h_f = ht. of W.T. from M.S.L at same position

h_s = ?

h_s = depth of interface below M.S.L.

Interface → equilibrium → water isn't moving

Pressure head at W.T. is zero.

$$(h_f + h_s)\gamma_f = \gamma_s h_s$$

$$h_s = \frac{h_f \gamma_f}{\gamma_s - \gamma_f} = \frac{h_f S_f}{S_s - S_f} = \frac{h_f S_f}{S_s - S_f}$$

$$= \frac{h_f}{\frac{\gamma_s}{\gamma_f} - 1}$$

| S → specific gravity

Date: / /

Usually, $S_s = 1.025$

$$h_s = \frac{h_f (1)}{(1.025 - 1)}$$

$= 40 h_f \longrightarrow (3)$

Let's say, $h_f = 3m$,

$\Rightarrow h_s = 120m \rightarrow$ From eq^n - (3)

Suppose $\rightarrow$ $h_f \overset{\text{becomes}}{=} 2m$ after pumping

h_s becomes 80

$\Rightarrow$ $\boxed{\text{For a 1m drop in fresh water, there'll be a 40m decrease in salt-water}}$

Hence, salt-water can infiltrate the well.

~~Ka~~ ∴ careful ~~ud~~ while pumping.

This process ~~is~~ called upconing. As we lower the W.T., salt-water goes up.

~~The mon~~

Coastal areas (A.P., Chennai)

Technique - put barriers → not effective

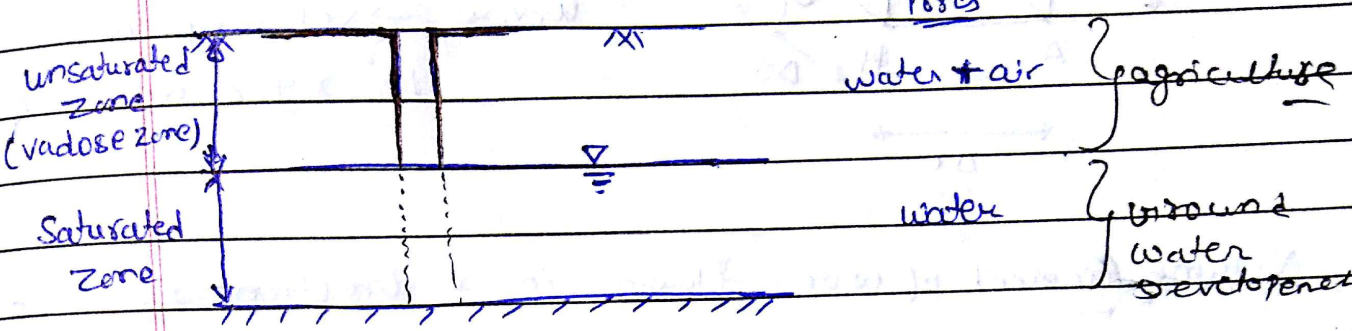
→ Difficult to solve using ~~a~~ differential eq^{n}s ∵ multi-phase (diff. densities)

Write separate D.E.s → coupled eq^{n}s (B.S.)

7/4/17

Flow Through unsaturated Zone

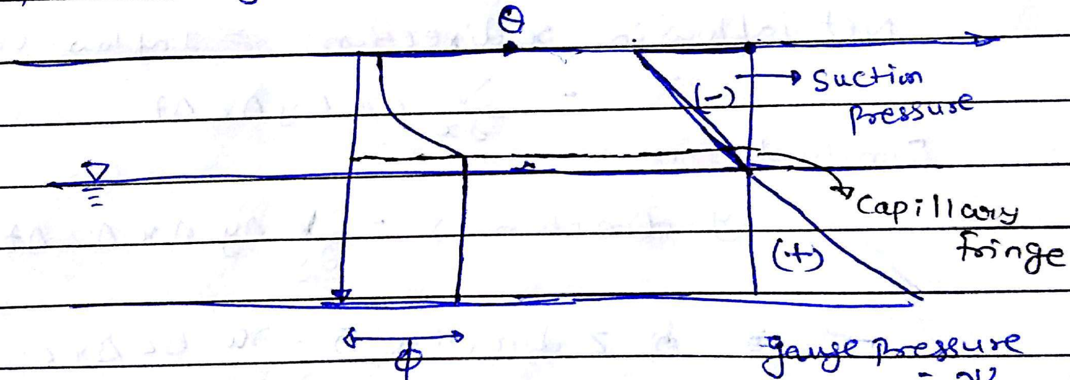
✗ ~~Important topic for Groundwater Development~~



* We have studied the flow through saturated Zone

θ = moisture content = $\dfrac{\text{Volume of water}}{\text{volume of soil}}$

ϕ = Porosity



Characterstics of unsaturated zone

1) −ve Pressure
2) $\theta \le \phi$
3) * Hydraulic Conductivity for a given soil in unsaturated zone depends on suction pressure.

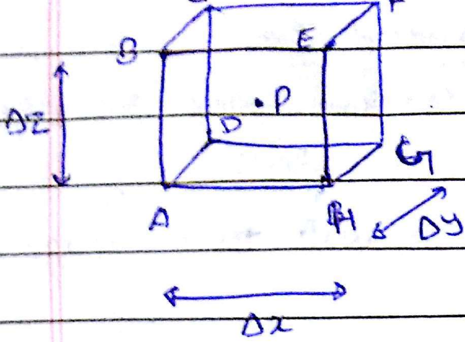
   $K = f(\text{suction Pressure, soil}) = f(\psi)$

   whereas, in saturated zone K is Constant for a given soil.
4) θ is a function of suction Pressure

   $\theta = f(\psi)$

ψ = suction Pressure = gauge pressure.

Date : ___/___/___



P = centre of element

at time Δt, θ is initial moisture content

$u, v, w \rightarrow$ velocity at P in x, y, z direction

Volume ~~Amount~~ of water flowing in x direction ~~from~~ to ABCD in Δt time

$$\Rightarrow \left(u - \frac{\partial u}{\partial x} \cdot \frac{\Delta x}{2}\right) \Delta y \, \Delta z \, \Delta t$$

flowing out through EFGH in x direction

$$= \left(u + \frac{\partial u}{\partial x} \cdot \frac{\Delta x}{2}\right) \Delta y \, \Delta z \, \Delta t$$

Net inflow in x direction = Inflow − outflow

$$= -\frac{\partial u}{\partial x} \Delta x \, \Delta y \, \Delta z \, \Delta t$$

Similarly in

y direction $\Rightarrow -\frac{\partial v}{\partial y} \Delta y \, \Delta x \, \Delta z \, \Delta t$

z ~~'&'~~ z direction $\Rightarrow -\frac{\partial w}{\partial z} \Delta z \, \Delta x \, \Delta y \, \Delta t$

Net Inflow in element from all direction

$$\Rightarrow -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) \Delta x \, \Delta y \, \Delta z \, \Delta t$$

$\Rightarrow$ change in storage is due to change in moisture content

$$\Rightarrow (\Delta \theta)(\Delta x \, \Delta y \, \Delta z)$$

$\downarrow$ change in moisture content in time Δt

$$-\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{\partial \theta}{\partial t}$$

K_x & K_y & K_z unsaturated hydraulic conductivity.

Date

Governing equation

$$-\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = \frac{\partial \theta}{\partial t} \longrightarrow (1)$$

$u, v, w, \theta \rightarrow$ 4 unknown

$$u = -K_x \frac{\partial h}{\partial x} \Rightarrow -K_x(\Psi)\frac{\partial h}{\partial x}$$

→ ~~Total head~~

K_x is function of Ψ

$$h = \Psi + z + \cancel{\frac{V^2}{2g}}^{\,0} \quad = \text{Total Head}$$

$$\boxed{h = \Psi + z} \quad \frac{\partial h}{\partial x} = \frac{\partial \Psi}{\partial x}\,;\ \frac{\partial h}{\partial y} = \frac{\partial \Psi}{\partial y}\,;\ \frac{\partial h}{\partial z} = \frac{\partial \Psi}{\partial z} + 1$$

Similarly, $v = -K_y \frac{\partial h}{\partial y} = -K_y(\Psi)\frac{\partial h}{\partial y}$

$$w = -K_z \frac{\partial h}{\partial z} \Rightarrow -K_z(\Psi)\frac{\partial h}{\partial z}$$

Put u, v, w in eqn (1)

$$K_x, K_y, K_z = f(\Psi)$$

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = \frac{\partial \theta}{\partial t}$$

Richards equation

~~non linear equation~~

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial \Psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial \Psi}{\partial y}\right) + \frac{\partial}{\partial z}\left[K_z\left(\frac{\partial \Psi}{\partial z} + 1\right)\right] = \frac{\partial \theta}{\partial t}$$

$\Psi, \theta \rightarrow$ 2 unknown

+1 → means gravity के कारण flow even Ψ are same for different z. still there will be a flow.

non linear equation

$$\frac{\partial \theta}{\partial t} = \frac{d\theta}{d\Psi}\frac{\partial \Psi}{\partial t}$$

$\frac{d\theta}{d\Psi} \rightarrow C$

$$\frac{\partial \theta}{\partial t} \Rightarrow C\frac{\partial \Psi}{\partial t}$$

C : Soil moisture capacity

Maths

ex:- $4\frac{\partial^2 h}{\partial x^2} + 3\frac{\partial h}{\partial x} + 5 = 0$ (linear)

$4x\frac{\partial^2 h}{\partial x^2} + 3x^3\frac{\partial h}{\partial x} + 5x^4 = 0$ (quasi linear)

$4h\frac{\partial^2 h}{\partial x^2} + 3h^3\frac{\partial h}{\partial x} + 5h^4 = 0$ (non linear)

x : independent variable

$$A\frac{\partial^2 h}{\partial x^2} + B\frac{\partial h}{\partial x} + C = 0$$

A, B, C constant = linear eqn

A, B, C = f(h) then it is non linear eqn]

A, B, C = f(x) quasi linear

Date : ___/___/___

Constitutive Relationship

we should know, $\theta - \psi$ (Relationship)

$K - \psi$

$\downarrow$ Soil-Moisture Characteristic = Soil-Retention Relationship

$$K = K_{sat} f(\psi)$$

K: Unsaturated Hydraulic Conductivity

K_{sat}: Saturated Hydraulic Conductivity

12/4/17

Richards equation

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial \psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z\left(\frac{\partial \psi}{\partial z} + 1\right)\right)$$

$$+ q_r - q_p = C(\psi)\frac{\partial \psi}{\partial t}$$

q_r: source, q_p: sink

$$C = \frac{d\theta}{d\psi} \qquad K_x, K_y, K_z = f(\psi)$$

for most of the field's ~~equ~~ Problems vertical flow is dominant = ~~q_r, q_θ~~, q_p is very less ~~than~~ ~~vertical~~ flow ~~so we~~ can neglect

~~Vertical~~

$$\frac{\partial}{\partial z}\left[K_z\left(\frac{\partial \psi}{\partial z} + 1\right)\right] + q_r - q_p = C\frac{\partial \psi}{\partial t}$$

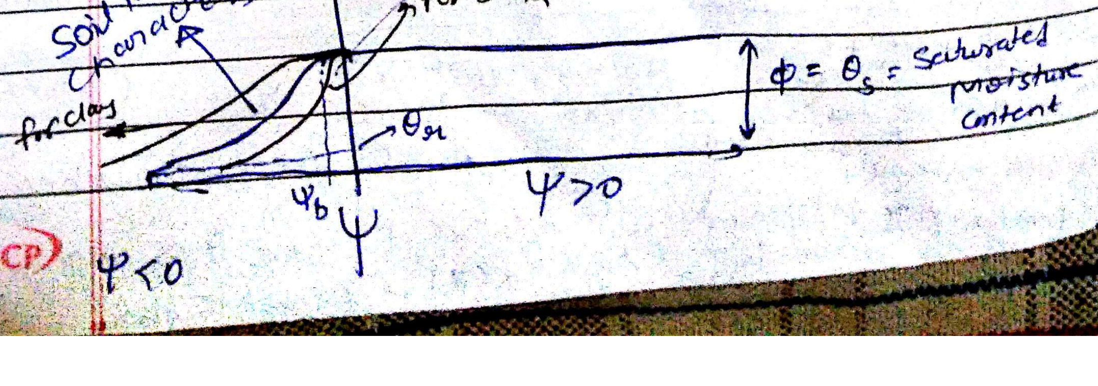
To solve above equation we required Constitutive Relationship

$\theta - \psi$ Relationship (Soil-Moisture or Soil Retention)

$K - \psi$

a) $\theta - \psi$ (SMC) Soil Moisture Characteristics

(SMC) $\Rightarrow$

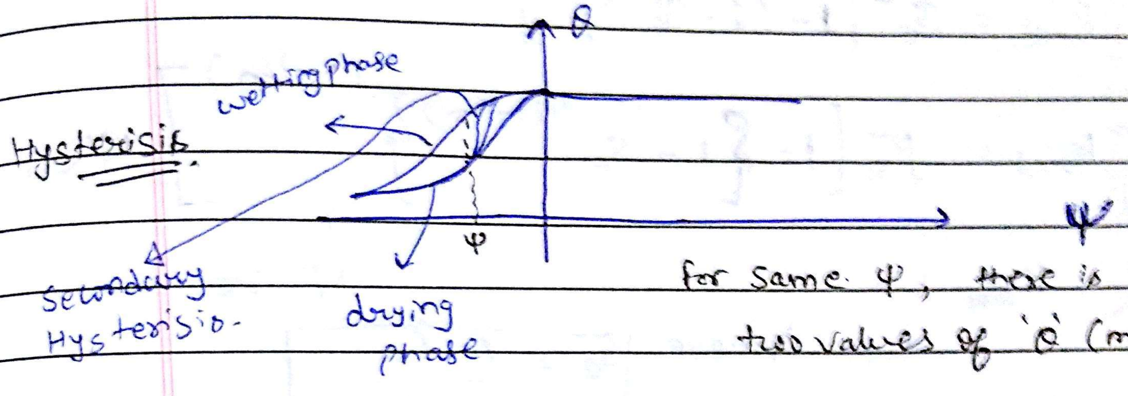


Date : ___/___/___

Ψ_b = Bubbling Pressure
(~~Below which~~ (Above which Air can't enter in soil ~~pores~~)

θ_r = Residual moisture

Hysterisis



for same ψ, there is two values of 'θ' (moisture content)

Most popular Relationship (Van Genuchten) (1980)
(θ−ψ)

S_e = Effective saturation

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \left[\frac{1}{1+(\alpha\psi)^n}\right]^{(1-\frac{1}{n})} \quad \forall \psi < 0$$

θ v/s ψ

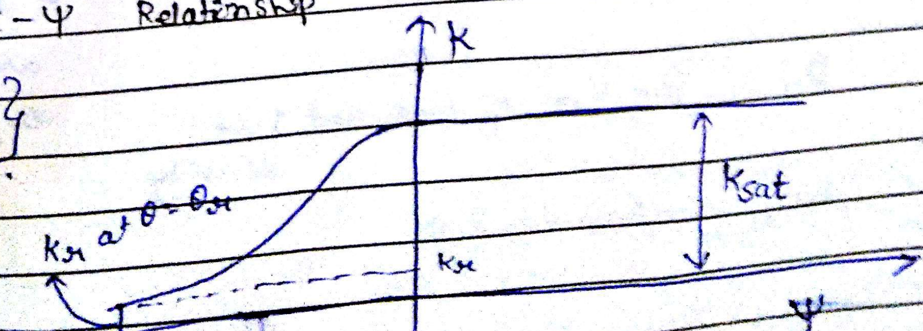
$$\begin{cases} \theta = \theta_r + (\theta_s - \theta_r)\left[\frac{1}{1+(\alpha\psi)^n}\right]^{(1-\frac{1}{n})} & \forall \psi < 0 \\ \theta = \theta_s & \forall \psi \geq 0 \end{cases}$$

Parameters, $\theta = f(\psi, \theta_r, \theta_s, \alpha, n)$

$$\boxed{C = \frac{d\theta}{d\psi}}$$

b) K−ψ Relationship

when $\theta = \theta_r$, $K = 0$



* for Practical Problem, Hysterisis is marginal in K−ψ Relationship
So, we neglect that.

$K = K_{rel} K_{sat}$ $\quad$ K_{rel} = Relative Hydraulic Conductivity

$$\boxed{0 \le K_{rel} \le 1}$$

$$K_{rel} = S_e^{1/2}\left[1-\left(1-S_e^{\frac{1}{1-1/n}}\right)^{(1-1/n)}\right]^2$$

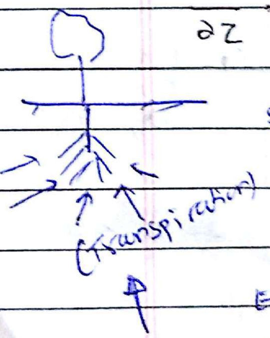
$$\begin{cases} K_{rel} = \sqrt{S_e}\left[1-\left\{1-S_e^{1/(1-1/n)}\right\}^{(1-1/n)}\right]^2 & \psi < 0 \\ K_{rel} = 1 & \psi \ge 0 \end{cases}$$

where $\boxed{S_e = \dfrac{\theta-\theta_r}{\theta_s-\theta_r}}$

So, $\boxed{K = f(\psi, \theta_s, \theta_r, n)}$

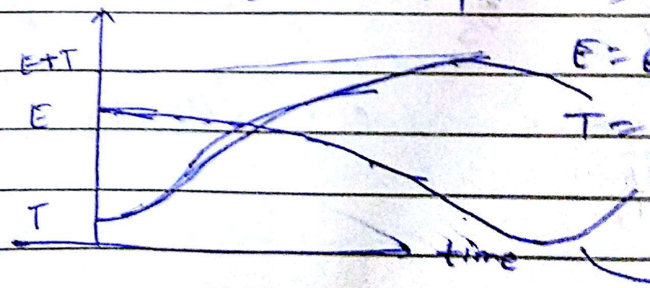
* Richards

$$\frac{\partial}{\partial z}\left\{K_z(\psi)\left(\frac{\partial \psi}{\partial z}+1\right)\right\} + \overset{\text{source}}{\theta_r} - \overset{\text{sink}}{\theta_p} = C\frac{\partial \psi}{\partial t}$$



Sink ~~Term~~ = Roots takes water

~~Heads~~: $\left\{\text{evapotranspiration} - \text{evaporation}\right\}$ = sink = Roots of जितना water लिया है



E = Evaporation

T = Transpiration

→ bc fruits and leaves are falling

So, EP, Tp

θ_p = function of time and root density

Irrigation Requirement