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En. No. - 1311306 3

Sub. - Ground Water Engg.

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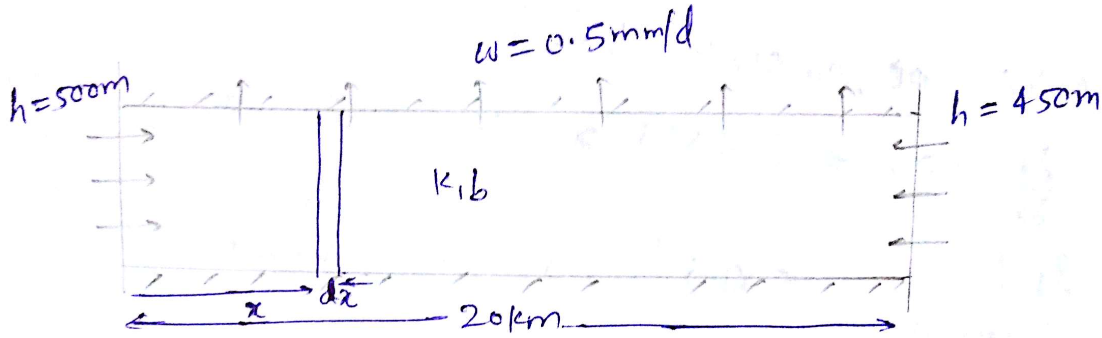
(1) ⇒



(a) $\because \quad Kb\dfrac{\partial^2 h}{\partial x^2} - w = 0$

$$Kb\frac{\partial^2 h}{\partial x^2} = w \Rightarrow \frac{\partial^2 h}{\partial x^2} = \frac{w}{T}$$

$$\frac{\partial h}{\partial x} = \frac{w}{T}x + C_1$$

$$h = \frac{w}{T}\cdot\frac{x^2}{2} + C_1 x + C_2$$

given, $w = 0.5\,mm/d$, $T = 1000\,m^2/d$

$= 5\times10^{-4}\,m/d$

$\therefore$ at $x = 0$, $h = 500\,m$, & $x = 20{,}000\,m$; $h = 450\,m$

$C_2 = 500$, $C_1 = -75\times10^{-4} = -7.5\times10^{-3}$

so, $\left[h = 2.5\times10^{-7}x^2 - 7.5\times10^{-3}x + 500\right]$

$$\left.\frac{\partial h}{\partial x}\right|_{at\,x=0} = 5\times10^{-7}\times0 - 7.5\times10^{-3} = -7.5\times10^{-3}$$

$$\left.\frac{\partial h}{\partial x}\right|_{at\,x=20Km} = 5\times10^{-7}\times2\times10^{4} - 7.5\times10^{-3}$$

$$= 2.5\times10^{-3}$$

Inflow rate (at $x = 0$ km) $= -1000 \left(\frac{m^2}{d}\right) \times (-7.5 \times 10^{-3})\, m \times 10^3 = 7500\ m^3/d/km$

Inflow rate (at $x = 20$ km) $= 1000 \left(m^3/d\right) \times (2.5 \times 10^{-3})\, m \times 10^3 = 2500\ m^3/d/km$

(b) for mini. peizometric head.

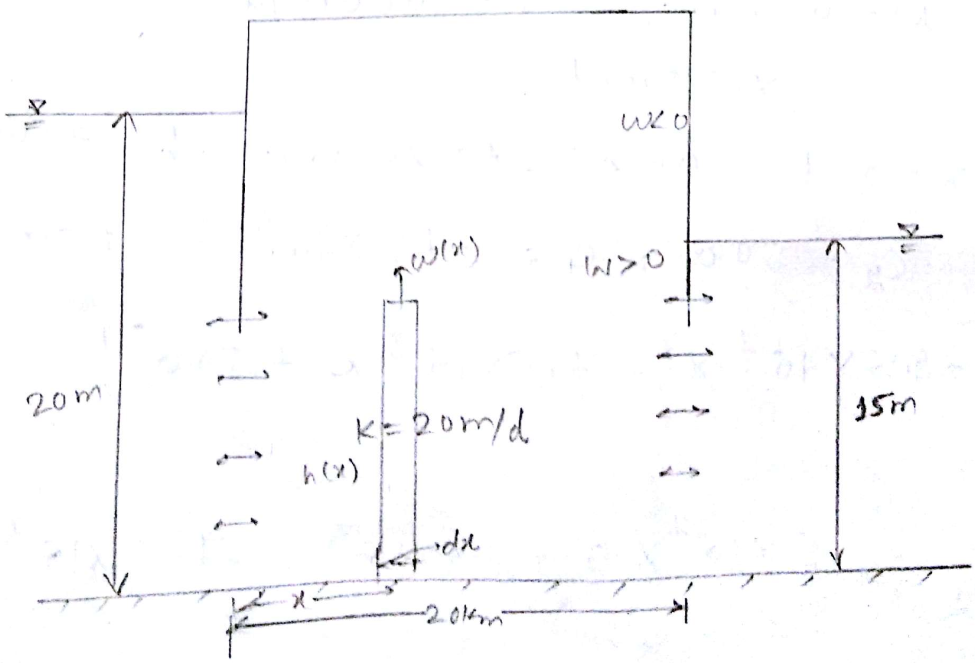
$\frac{\partial h}{\partial x} = 0 \Rightarrow 5 \times 10^{-7} x - 7.5 \times 10^{-3} \Rightarrow \underline{x = 15\ Km}$

$\left.\frac{\partial^2 h}{\partial x^2}\right|_{at\ x=15} = 5 \times 10^{-7} > 0$, so minima.

$h\Big|_{at\ x = 15\ Km} = 2.5 \times 10^{-7} \times (15 \times 10^3)^2 - 7.5 \times 10^{-3} \times (15 \times 10^{+3}) + 500$

$\left[h_{min.} = 443.75\ m\right]$

(2)→



$$\frac{\partial}{\partial x}\left(\frac{h\,\partial h}{\partial x}\right) = \frac{\omega}{K}$$

$$h\frac{\partial h}{\partial x} = \frac{\omega}{K}x + C_1$$

$$\frac{h^2}{2} = \frac{\omega}{K}\frac{x^2}{2} + C_1 x + C_2$$

$$h^2 = \frac{\omega}{K}x^2 + C_1 x + C_2$$

at $x=0$, $h = 20m$, $x = 20000m$, $h = 15m$

$$C_2 = 400$$

$$225 = \frac{\omega}{20} \times (2\times10^4)^2 + C_1 \times (2\times10^4) + 400$$

$$C_1 = \frac{-175}{2\times10^4} - \omega\times10^3$$

$$\frac{\partial(h^2)}{\partial x} = \frac{2\omega}{K}x + C_1 = 0 \Rightarrow x = -\frac{KC_1}{2\omega}$$

$$h^2 = \frac{\omega}{K} \times \frac{K^2C_1^2}{4\omega^2} - \frac{KC_1^2}{2\omega} + C_2$$

$$h^2 = \frac{-KC_1^2}{4\omega} + C_2$$

Now, for $\omega > 0$, $h = h_{min}$.

$$5^2 = \frac{-KC_1^2}{4\omega} + 400$$

$$\omega = \frac{KC_1^2}{4\times375} = \frac{K}{4\times375}\times\left[\frac{175}{2\times10^4} + \omega\times10^3\right]^2$$

$$75\omega = 7656.26\times10^{-8} + \omega^2\times10^6 + 17.5\omega$$

$$\omega = 56.13\times10^{-6}\ m/day. \quad @ \quad 1.365\times10^{-6}\ m/day$$

for $\omega < 0$, $h = h_{max}$.

$$(23.5)^2 = \frac{-KC_1^2}{4\omega} + 400$$

$$-30.45\omega = \left[\frac{175}{2\times10^4} + \omega\times10^3\right]^2$$

$$10^6\times\omega^2 + 47.95\,\omega + 7656.25\times10^{-8} = 0$$

$$\omega = -46.29\times10^{-6}\ m/d,\ \text{or}\ -1.653\times10^{-6}\ m/d$$

Permissible range of W for given max. & min head.

$$\left[-46.29\times10^{-6}\ m/day < W < 56.13\times10^{-6}\ m/day\right]$$

(b) ⇒ −ve sign of ω indicate recharge

Max. water that can be taken out $= 1.5\ mm/d + 56.13\times10^{-6}\ m/d$ (↑ vertical recharge)

$= 1.556\ mm/d$

Let max. no. of well, N_{max}.

Water coming out per km width $= N_{max}\times 50\times8\ m^3/d$

$= 400 N_{max}\ m^3/d$

Max water per km width $= 1.556\times10^{-3}\ m\times20\times10^3\times10^3\ m/d$

$= 3.1122\times10^4\ m^3/d$

N_{max} per km width

$$= \frac{3.1122\times10^4}{400}$$

$= 77.8$ wells / km width

So, ~~ ~~ Max. no. of well $= 77$ per km width.

$$= 1.5 \frac{mm}{d} - 4.629 \times 10^{-5} \frac{m}{d}$$

$$= 1.45371 \times 10^{-3} \frac{m}{d}$$

Similarly,

$$N_{min} = \frac{1.45371 \times 10^{-3} \times (20 \times 1000)(1000)}{50 \times 8}$$

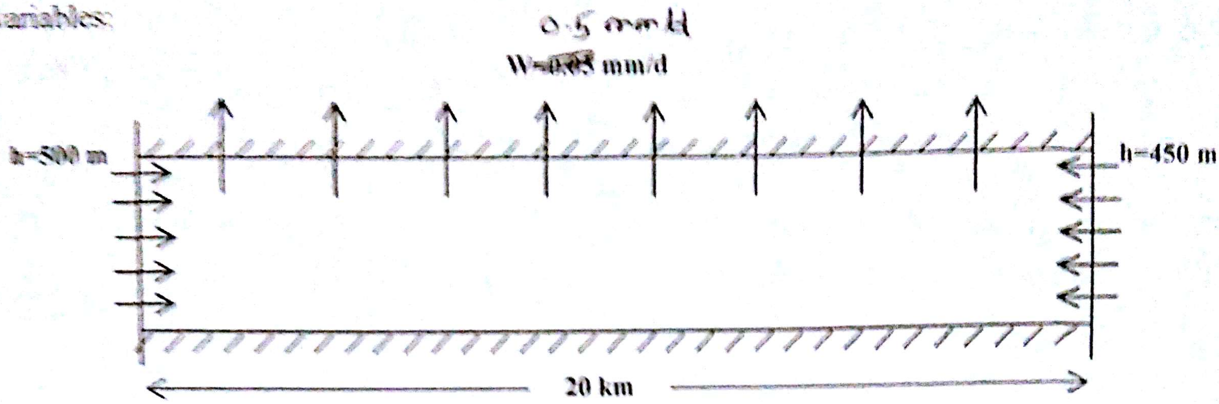
$$N_{min} = 72.68$$

$$\boxed{N_{min} \simeq 73}$$

CE 535 /423 – Groundwater Engineering

TUTORIAL – 3

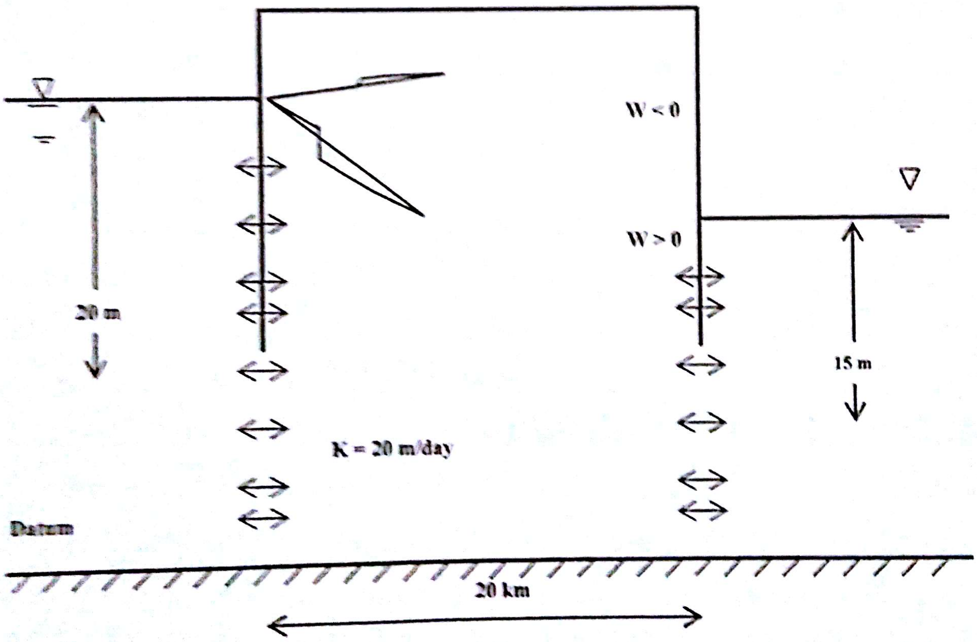
Q1. Consider the following 1-D flow in a confined aquifer and compute the following state variables:



a) Lateral inflow rate (m^3/day/km) at the two ends.

b) Minimum peizometric head. Given T=1000m^2/d.

Q2. Consider the following 1-D flow



a) Maximum & Minimum permissible water table elevations are 23.5 m & 5 m respectively above the datum. Compute the corresponding permissible range of W. Also compute the corresponding stream aquifer's flow rate.

b) Given vertical recharge in the area is 1.5 mm/day. Compute the maximum and minimum permissible number of wells/km width in the area, taking unit discharge of well as 50 m^3/hr and average daily running hours as 8.